LONDON MATHEMATICAL SOCIETY

Use and Misuse of Information and Computer Technology in the Teaching of Mathematics at HE Institutions

Position Statement

Outline

Over the last two decades, the academic mathematics community has accumulated significant collective experience in use of Information and Computer Technology (ICT) in the teaching of mathematics (which for this report includes pure maths, applied maths, statistics and operational research) at HE institutions (at both undergraduate and postgraduate level) as well as service courses for engineering and science students. The development and adoption of new tools for learning and teaching mathematics was primarily driven by absorbtion of technology from mathematical research. Computational, ICT and word-processing tools such as LaTeX, MATLAB, R, MATHEMATICA and MAPLE have been originally created for research but happened to be excellently suited to the teaching and presentation of mathematics. As a rule, mathematics lecturers themselves have led the way in the innovative use of ICT in teaching mathematics, and many mathematics courses had a significant ICT and computational element well before its uses in other disciplines.

It needs to be emphasised that MATLAB, MAPLE, MATHEMATICA and statistics packages such as SPSS and R are not just toys for learning they are professional research tools; mastering them is a valuable transferable skill for graduates seeking employment in mathematics-intensive industries.

However, there are also significant threats to mathematics teaching presented by the misuse of ICT. In particular, a number of obstacles to efficient use of ICT in university level teaching of mathematics arise from the restructuring of ICT provision in universities, which, in many universities, is now highly centralised and deprives individual departments of their say on ICT policy.

This report is addressed not only to our mathematicians but also to our non-mathematician colleagues in British universities. We survey the use of ICT for teaching mathematics, explicate the position of mathematicians and make some general policy recommendations (see the next page). We refrain from giving to our colleagues specific advice on teaching. Dissemination of good practice (and the accumulated considerable positive experience of the mathematics community in the use of ICT in teaching) is best done via other channels.

Finally, we emphasise that the recommendations made in this document apply only to higher education. Mathematics teaching in schools and Further Education colleges is taking place in a different environment and is likely to require different approaches.

Summary of recommendations

1. **Selectivity:** The specific cognitive nature of mathematics and wide diversity of content and aims of university level mathematics courses dictate a highly selective approach to choice of software and ICT solutions used in teaching and learning. Tools useful in one course might be completely unsuitable for another course in the same year of the same degree programme; solutions usefully applied in postgraduate level courses could be harmful in undergraduate teaching.
2. **Costs involved:** There is no evidence that computerisation of mathematics teaching and learning saves time and money. As a rule, successful use of ICT in teaching mathematics relies on large amounts of unpaid work of individual teachers in addition to their usual teaching and research workload. There are obvious dangers in basing a large scale policy on such a fragile foundation.
3. **Delivery:** Tools such as MATLAB, computer algebra packages and R have a proven record of enhancing suitable specific courses when used appropriately. However they are no substitute for traditional face-to-face teaching.
4. **Distance learning:** This should only be used if there is significant personal tutorial support available to students.
5. **Virtual Learning Environments:** So far, they do not live up to their promise. We recommend (limited) use only of those products (such as Moodle) which support mathematical notation (in particular LaTeX).
6. **Word-processing:** We need to reject ICT products not suited to writing, presenting and processing mathematical texts. LaTeX is a recognised solution, and students should be encouraged, and where possible taught, to use LaTeX to present mathematics in reports and projects.
7. **Computer aided assessment:** Existing tools suffer from inadequate student interfaces which still have limited facilities for a natural and intuitive entry of mathematical formulae.
8. **On-line resources:** Some excellent resources are already available. We need to give more support to Open Source textbooks, software and courseware.
9. **Visually impaired students:** Special consideration needs to be paid to the needs of visually impaired students who face particular difficulties when accessing mathematics. Tools in LaTeX need to be developed to account for the needs of such students.

We now detail the evidence behind these summary recommendations.

1 Selectivity: choice of tools for direct delivery of mathematics

Mathematicians were responsible for the invention of the computer, and computation plays an essential role in modern mathematics, both pure, applied and statistics. Courses in such areas as numerical analysis, optimisation, linear algebra, statistics and discrete mathematics rely very heavily on computation, and computation plays an increasingly important role in such diverse areas as number theory, logic, differential equations and mechanics. Computation is important in modern mathematics research and many (if not most) mathematics undergraduates will go on to careers which make a substantial use of ICT.

Specialised tools have been developed by mathematicians to aid in the teaching of both undergraduate and graduate mathematics courses. For example MATLAB is a useful tool for the teaching of linear algebra, differential equations and signal processing, and is also an excellent general purpose graphical engine for visualising mathematical surfaces. Similarly the package R is a crucial component of most statistics courses. Other useful mathematical software includes MAPLE, MATHEMATICA and SPSS.

One point that must be made however, is that ICT supported teaching only works well if the right tools are used. For example, if you want to solve a differential equation numerically and present the solution graphically, then you should use MATLAB. Our colleagues from some universities reported that they were pressurised to do the same using standard spreadsheets; this approach is at best highly inefficient and at worst extremely confusing for the students.

However, whilst these teaching and learning techniques are very useful, and can significantly illuminate a course, they are still no substitute for the more direct, face-to-face and usually board based, methods used to teach mathematics.

This point was made strongly in the LMS Teaching Position Statement [[1]] and need not be repeated here. Suffice to say, that when developing complex arguments, especially when proving elaborate theorems, the students need to see the mathematics being developed in front of them, and must be able to see many lines of argument. This simply cannot be done effectively with the rather restricted delivery afforded by ICT (whether it is a Power Point presentation or the use of a visualiser), and a presentation using large visible boards is much more effective in this regard.

It is also important to note that a tool such as MATLAB is only effective when

[1] `http://www.lms.ac.uk/sites/default/files/Mathematics/Policy_repors/2010teaching_position_statement.pdf`.

used as *part* of an overall teaching environment. In particular

(i) the students must have enough mathematical background in order to understand the results that they are seeing and to reject as incorrect the results of an incorrect programme,

(ii) most (if not all) mathematics students have signed up to do a mathematics degree because they like, and want to do, mathematics. Pressing buttons on a keyboard is no substitute for actually doing mathematics and learning a mathematical argument.

We need to address a growing cultural gap affecting students' expectations of ICT. For example, there is evidence from several universities that the students, whilst recognising the importance of using MATLAB in numerically intensive courses, do not especially like the experience of actually using it. The principal reason for that is the need to use the command line interface (and even write executable scripts)—tasks completely absent in the mainstream computer use culture.

2 Costs involved

There is no evidence that computerisation of mathematics teaching and learning saves time and money. As a rule, successful use of ICT in teaching mathematics relies on large amounts of unpaid work of individual teachers in addition to their usual teaching and research workload. There are obvious dangers in basing a large scale policy on such a fragile foundation.

Like mastering music instruments, teaching / learning mathematics is best done one-to-one, or in a small group. Large class lectures are an unhappy compromise with economic necessity. From a pedagogical point of view, the right alternative to a large class lecture is not streaming-on-demand of video recordings; the true alternative is a small class lecture. Unfortunately, we have to accept that this alternative in most cases is financially infeasible. Collaborative on-line small groups provide some interesting possibilities, but students themselves insist that ICT should be a supplement, not a replacement of the face-to-face teaching:

> Motion 306, passed at the April 2010 NUS National Conference states that: [...]
>
> 4. The provision of e-learning should be utilised as a tool for learning, in all institutions, but that should not merely be used as a method of reducing costs and should be in conjunction with, not instead of, other face-to-face teaching methods.
> 5. Technology should complement good teaching, allowing students to benefit from the additional value of e-learning but should not

> be used as a substitute for face-to-face contact and good teaching. [[2]]

The LMS shares this position of our students.

This report addresses various aspects of teaching to large classes: delivery, communication, assessment. We emphasize, however, that large class teaching is already, by default, under-resourced teaching. It is futile to expect further savings brought by use of expensive technology.

3 Delivery and face-to-face teaching

Studies of students' attitudes to ICT already exist, and [[3]] provide a useful survey. A recent report from the National Union of Students expresses a summarised students' opinion in a very direct and unambiguous way.

Students want to have choice and want to be in control:

> Students prefer a choice in how they learn—ICT is seen as one of many possibilities, alongside part-time and traditional full-time learning, and face-to-face teaching. [[4]]

> Students could see some advantages to an e-learning approach. If it were presented as an option, as opposed to an obligation, it would avoid onerous undertones. [[5]]

Another point is that content matters for students more than delivery:

> Participants expressed concerns over "surface learning" whereby a student only learns the bare minimum to meet module requirements—this behaviour was thought to be encouraged by ICT: students can easily skim-read material online, focusing on key terms rather than a broader base of understanding. [[6]]

[2] Student perspectives on technology—demand, perceptions and training needs. Report to HEFCE by NUS 2010, p. 18. `http://www.hefce.ac.uk/pubs/rdreports/2010/rd18_10/rd18_10.pdf`.

[3] Learner acceptance of on-line learning and e-learning, `http://wiki.alt.ac.uk/index.php/Learner_acceptance_of_on-line_learning_and_e-learning`.

[4] Student perspectives on technology—demand, perceptions and training needs. Report to HEFCE by NUS 2010, p. 3. `http://www.hefce.ac.uk/pubs/rdreports/2010/rd18_10/rd18_10.pdf`.

[5] *ibid.*, p. 5.

[6] *ibid.*, p. 5.

4 Distance learning of mathematics

A key feature to consider here is the special nature of mathematics as a subject. Its difficulty and the impenetrability of certain subjects the first time they are met (for example limits) and also the way that students can get completely stuck on problem solving, means that it is essential that most students of mathematics have close, personal, support in their learning experience. Distance learning can only operate effectively if it used in this context.

There is already some significant experience in the use of distance learning methods to teach mathematics. A notable recent use of this has been in the EPSRC *Taught Course Centres*, such as, for example, the MAGIC consortium. In these centres, leading research universities work together to deliver a focused programme of graduate level teaching through a video conferencing approach. There are four important features of graduate teaching which make this approach both necessary and viable.

- Firstly, graduate teaching is specialized and is delivered to *small groups*. If such small groups were on their own in a single university then it might be simply inviable in resources to teach them, however, by joining together then the classes across several universities one gains a critical mass and the classes then become resource effective.
- Secondly, the classes are delivered by clusters of universities that work together as a team on an equal footing.
- Thirdly, graduate students by their very nature are able and motivated students.
- Fourthly, all of the students on the course will have extensive back-up and support *at their own universities*.

Other areas where distance learning has proved to be effective, precisely because the four points above apply, are (i) engagement of universities with training in industry (ii) the MOTIVATE programme which uses video conferencing to link HE with schools.

But, none of these four points above apply to *undergraduate teaching* where classes are large. In a scenario promoted by some politicians one university would deliver distance learning to a series of much smaller institutions. The main danger here is that at the receiving end of this delivery system we are likely to find a significant number of weakly prepared or unmotivated students and students without direct support at their home base. The experience of the Open University, which has been using distance learning courses for a long time, is that they only work because the students on them are highly motivated and have access to tutors and extensive resource materials.

5 Virtual Learning Environments

Well used VLEs can be effective in enhancing the delivery of mathematics courses, especially when classes are large and the lecturer needs a simple way of communicating with the whole student group (such as providing additional resources and challenges, and giving news about the course). However, by and large, they have not really lived up to their promise in this regard. To be effective for teaching and communicating mathematics it is **essential** that such VLEs are able to cope with mathematical notation, graphs and figures. Some products such as *Moodle* can, others cannot. Enforcing the use of a product unable to deal with mathematical notation is a grotesque waste of time for all concerned. They simply cannot be used to teach mathematics and they get in the way of other products which are much more useful.

It is interesting to note that Moodle, which allows for mathematical notation, is free and open source, whereas other proprietary VLEs make no such provision.

6 Presenting mathematics: word-processing and presentation tools

One of the most successful tools for the presentation of mathematics is LaTeX. Initially developed by Knuth in the 1970s it is now the de-facto standard for scientific publications. This document is written in LaTeX. The huge majority of mathematics papers are written in LaTeX, and the greater majority of computer based mathematical presentations at conferences and seminars are delivered using a PDF file created by LaTeX. Note that LaTeX is also free and open source.

There is simply no substitute for LaTeX for producing complex mathematical expressions such as

$$\Gamma(z) = \lim_{n\to\infty} \frac{n!\, n^z}{z(z+1)\cdots(z+n)} = \frac{1}{z}\prod_{n=1}^{\infty} \frac{\left(1+\frac{1}{n}\right)^z}{1+\frac{z}{n}}. \tag{1}$$

LaTeX provides the most convenient mechanism for typesetting such equations, but also provides some of the best facilities for creating large structured documents. This includes setting out theorems and proofs, numbering equations, citing references, producing tables and arrays and importing graphics.

There are a number of separate processes when creating a high quality document which begin with *authorship*. Next comes *typesetting* in which the author literally types their document into a machine. This is followed by *processing* of the document and finally *publication*. Notice that traditional 'word processors' confuse typesetting and processing, possibly encouraging authoring at the same time. LaTeX, on the other hand, consciously separates them. This enables consistency to be easily and naturally imposed on a document, and for sections, equations, figures etc. to be numbered accurately. For example, in equation (1) above, the author does not type "(1)" when referring to it, but instead uses a tag such as `eqn1`. The actual number is automatically assigned, enabling equations to be inserted or removed with confidence that the numbering in the document will be accurate when finally processed. Much more seriously, the common practice in Word of selecting surface features (typeface, size, font weight, position on line, etc.) is at odds with the notion in LaTeX of specifying the *purpose*, such as "section heading". Word does have "styles" but few people understand them, fewer still use them consistently. By creating such a structured document, the different needs of readers can easily be accommodated by processing the document with different styles to suit *dyslexic* or partially sighted readers. (We will consider the case of severely visually impaired students in the final section.) Word processors confuse these issues which makes it very difficult to create high quality structured documents.

The commercially available alternatives do not come close to the quality of the presentation of mathematics available in LaTeX, their development still lags 30 years behind that of LaTeX. Institutional requirements which force mathematicians to use them are invitations to return back to the Stone Age. Much more seriously, these mathematical tools have been additions to the software which have not remained stable over time. LaTeX, on the other hand, has been remarkably stable, while still keeping pace with new publication formats such as PDF files, and technological innovations such as the need to embed active URLs within documents.

Increasingly students in mathematics have to produce project work as part of their assessment. For example, in most MMath or MSci courses a compulsory project forms a major part of the final years assessment. It is important that such students to produce a written project report and slides for a presentation and to do this they need the right tool for the job, namely LaTeX. However, it is fair to say that learning LaTeX requires some effort and practice. Thus we encourage universities to make provision of LaTeX courses for their undergraduates—such courses are already available in a number of universities.

7 Computer Aided Assessment of Mathematics

Assessment is a key issue in all subjects. The large classes already mentioned make rapid marking of students' work difficult. This poses a severe difficulty in the use of formative assessment. However, a partial solution to this problem is provided by Computer Aided Assessment (CAA).

Again, we wish to repeat that large class teaching is by default under-resourced teaching and that one should not expect money to be saved by use of CAA.

Automatic assessment is commonly associated with multiple choice questions (MCQ). Indeed, many existing generic CAA systems such as those provided centrally as institutional "learning environments" provide only types of interactions in which *potential answers provided by the teacher* are selected by students.

It is very difficult to write effective MCQ items and in many situations the teacher is essentially forced to "give the game away" by presenting these choices up front. The student then has only to select or verify rather than create.

In mathematics in particular the purpose of many questions is grotesquely distorted by using a MCQ since the difficulty of a reversible process is markedly altered in different directions. For example, solving an equation from scratch is significantly different than checking whether each potential response is indeed a solution. Expansion versus factorization of algebraic expressions, or integration versus differentiation are further examples. What many of these examples have in common is the difficulty of an *inverse operation* relative to the direct operation.

The strategic student does not answer the question as set, but checks each answer in reverse. Indeed, it might be argued that it is not just the strategic, but the *sensible* student, with an understanding of the relative difficulties of these processes, who takes this approach. This distortion subverts the intention of the teacher in setting the question, so that we are not assessing the skill we wish to assess. Hence the format renders the question *invalid*.

There are other problems with the MCQ format. Some authors go as far as saying MCQ tests *"favor the nimble-witted, quick-reading candidates who form fast superficial judgements"* and *"penalize the student who has depth, subtlety and critical acumen"*. [[7]] Further, it is claimed that the MCQ format itself has inherent gender bias [[8]]. To avoid these problems with the MCQ, and similar, question types mathematicians strongly prefer CAA systems which evaluates answers provided by students which consist of their own mathematical expressions. There is a long track record of this within the mathematics community, from the 1970s [[9]] until today.

There are many CAA systems in use by mathematicians. They share the need to display mathematical notation online, to respond in a sophisticated way to mathematical input from students. With decades of experience these systems are relatively common, although standards and common formats are yet to be agreed. Such systems see most use in methods based courses where the object of the exercise is to *obtain an answer* using a standard technique. They have less use in other courses where the purpose of the question is to *justify* or *criticize*. Nevertheless, mastery of lower-order technical skills is a precursor for progress at advanced levels. Repetitive practice of tutorial examples characterizes the traditional method of teaching, and here CAA is playing a very useful role quite widely.

One common difficulty is when institutions require mathematics students to use

[7] B. Hoffmann. *The tyranny of testing*. Crowell-Collier, 1962.

[8] P. Hassmén and D. P. Hunt. Human self-assessment in multiple choice. *Journal of Educational Measurement*, 31(2):149–160, 1994.

[9] See D. Sleeman and J. S. Brown, editors. *Intelligent Tutoring Systems*. Academic Press, 1982, for a survey.

very limited generic CAA systems which do not adequately support mathematics notation.

We need to understand, however, the unavoidable limitations of CAA: they are better suited for testing routine procedural skills rather than creative thinking and understanding of highly abstract concepts.

We should expect a pressure to switch to CAA not only in formative assessment and coursework tests, but also in course examinations. Indeed, experience shows that a formative CAA translates better to good exam results if the exams are set in the CAA format already familiar to students. There is a danger that if students see that the use of CAA for formative assessment helps to achieve desired test and exam results they are likely to make the CAA their learning tool of choice and ignore other forms of learning.

"Teaching to the test" is already a dangerous but underestimated trend that slowly erodes the fundamentals of mathematical education. The main danger associated with the CAAs is that their easy availability will increase the already existing pressure to "teach to the test"— and, which could happen to be a much worse outcome—"to teach to the *computerised* test". Paradoxically, the more successful a CAA the more harm it may bring to mathematics education in the long run.

8 On-line resources for mathematics

Mathematics, by its nature, is an open source phenomenon, with mathematical results and ideas freely available. We are now in the position of having open source textbooks (licensed by GNU) which have a functionality (such as global editing facilities) which make them very useful for mathematics teaching. A surprisingly large amount of free open source mathematical software is also available online. These promise a potential revolution in the manner in which open source electronic texts can present mathematics. We encourage and support the use and adoption of these for enhancing undergraduate teaching

9 ICT and visually impaired students

Visually impaired students have obvious difficulties in accessing a visually intensive subject such as mathematics. They not only need notes in Braille which adequately present mathematical formulae, but they need ways of interacting with graphical displays on computer screens. Limited provision exists at the moment for each. In particular, it is essential that LaTeX tools are developed for easy conversion of teaching materials into a format accessible to visually impaired students.

Some possibly useful solutions appear to be relatively straightforward from a technical point of view; for example, it appears natural to try to develop a mark-up language for embedding into LaTeX files that would provide creators of LaTeX files with tools for the creation and control of PDF tags in output PDF files (thus making tables and footnotes accessible to keystroke navigation) and for writing from LaTeX directly into the accessibility layer, making, for example, mathematical formulae readable by screen readers. One may think about something like

```
\[\int_0^1 2x^3 dx\]%
\readaloud{integral from zero to one of two "x" cube "dx"}
```

being converted into a pdf file which properly renders LaTeX on the screen, as

$$\int_0^1 2x^3 dx,$$

while the argument of `\readaloud` is being read aloud (without, of course, being shown on the screen).

One immediate difficulty is that there are no even universally accepted rules for reading complex mathematical formulae aloud. The LMS would welcome any project aimed at adding assistive functionality to LaTeX.

We note that there are many other issues concerning the access of disabled students to mathematics courses. These should most properly be the subject of a future position statement.

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