Citation for Julian Sahasrabudhe (Whitehead Prize)

Short citation

Dr Julian Sahasrabudhe of the University of Cambridge is awarded a Whitehead Prize for his outstanding contributions to Ramsey theory, his solutions to famous problems in complex analysis and random matrix theory, and his remarkable progress on sphere packings.

Long citation

Dr Julian Sahasrabudhe of the University of Cambridge is awarded a Whitehead Prize for his remarkable contributions to several areas of mathematics, including sphere packings, random matrices, complex analysis and Ramsey theory. His work has changed the landscape in each of these areas. His results are characterised by the most bold and daring proof schemes, which initially seem entirely unrealisable but which he succeeds in making work through raw power and incredible creativity.

Sahasrabudhe began his research in partition regularity, a deep and notoriously difficult area of Ramsey theory. He was able to combine two different operations, multiplication and exponentiation, in ways that had seemed completely out of reach. He showed, for example, that the system x, y, x^y is partition regular. Prior to this, nobody had had any methods that seemed at all plausible for attacking this question. He went on to generalise this enormously, revealing structure that had not even been hinted at before.

In analysis, Sahasrabudhe has worked on estimates for cosine polynomials. He has solved Littlewood's famous conjecture, that the number of roots of cosine polynomials such as $\cos a_1x + \cos a_2x + \ldots + \cos a_nx$ must go to infinity as the number of terms n increases. This problem had been open for 50 years, and had attracted a lot of attention. He gives a lower bound (in fact, an explicit lower bound) on the number of roots: basically it is $(\log \log \log n)^{1/2}$. This is an astonishing breakthrough, but perhaps even more impressive than the result itself is his method. What was missing from all earlier work was some sense of 'what do trigonometric polynomials with few roots look like?' It was known, via examples, that they tend to have strange properties, but nobody could ever prove anything positive about them. Sahasrabudhe succeeds in this: he manages to classify such polynomials. What he shows is a remarkable and unexpected fact: that they must 'correlate' with some easier-to-analyse functions.

Sahasrabudhe has also worked in complex analysis, on Littlewood polynomials. He has solved what was undoubtedly the biggest open problem in this area, a famous problem of Littlewood that asks if there are polynomials of degree n, with all coefficients ±1, such that the image of the unit circle is bounded both above and below by a multiple of $n^{1/2}$. Such polynomials are called 'flat Littlewood polynomials', and their existence has been investigated by many authors (including Littlewood and Erdős). Rudin and Shapiro, in the 1950s, constructed polynomials satisfying the upper bound, but they did not satisfy the lower bound. The usual belief has been that flat Littlewood polynomials do not exist. Indeed, computer searches by several authors, notably Odlyzko, pointed strongly in this direction.

Sahasrabudhe solves the problem. In joint work with Balister, Bollobás, Morris and Tiba he shows that, in fact, flat Littlewood polynomials do exist. The proof is an amazing and intricate blend of hard analysis (properties of particular trigonometric polynomials) and discrepancy theory.

Recently, Sahasrabudhe has returned to Ramsey theory. He has given the first exponential improvement on the Ramsey numbers R(s) in over 70 years. It was known that the Ramsey numbers grow at a rate that is at most 4^s, but nobody could improve on the '4'. This has been perhaps the central problem in all of Ramsey theory. Sahasrabudhe, in joint work with Campos, Griffiths and Morris, gives an upper bound of $(4 - ǫ)^s$ for a fixed positive constant ǫ.

Sahasrabudhe has also made extraordinary progress on sphere packings in high dimensions. The sphere packing problem asks for the densest packing of unit spheres in n-dimensional Euclidean space. There are constructions giving density $n/2^n$, and these have not been improved upon for a very long time, apart from by some constant factors. In joint work with Campos, Jenssen and Michelen, he gives a lower bound of log n times this. This is absolutely remarkable. What is fascinating is the method: whereas all previous attacks on the problem have been via deterministic packings, Sahasrabudhe introduces random ideas. But the scene has to be set very carefully indeed: it is only once the right framework is in place that the random method has any hope of getting anywhere.

Finally, Sahasrabudhe also has remarkable results in random matrix theory. Estimates on the singularity probability of sparse random 0-1 matrices were known, but bounds on the eigenvalues were lacking. In joint work with Sah and Sawhney, he is able to find the exact limiting spectral law. The proof is again an absolute tour-de-force, with breathtaking proof-schemes that seem impossible to carry through but which are then indeed carried through.