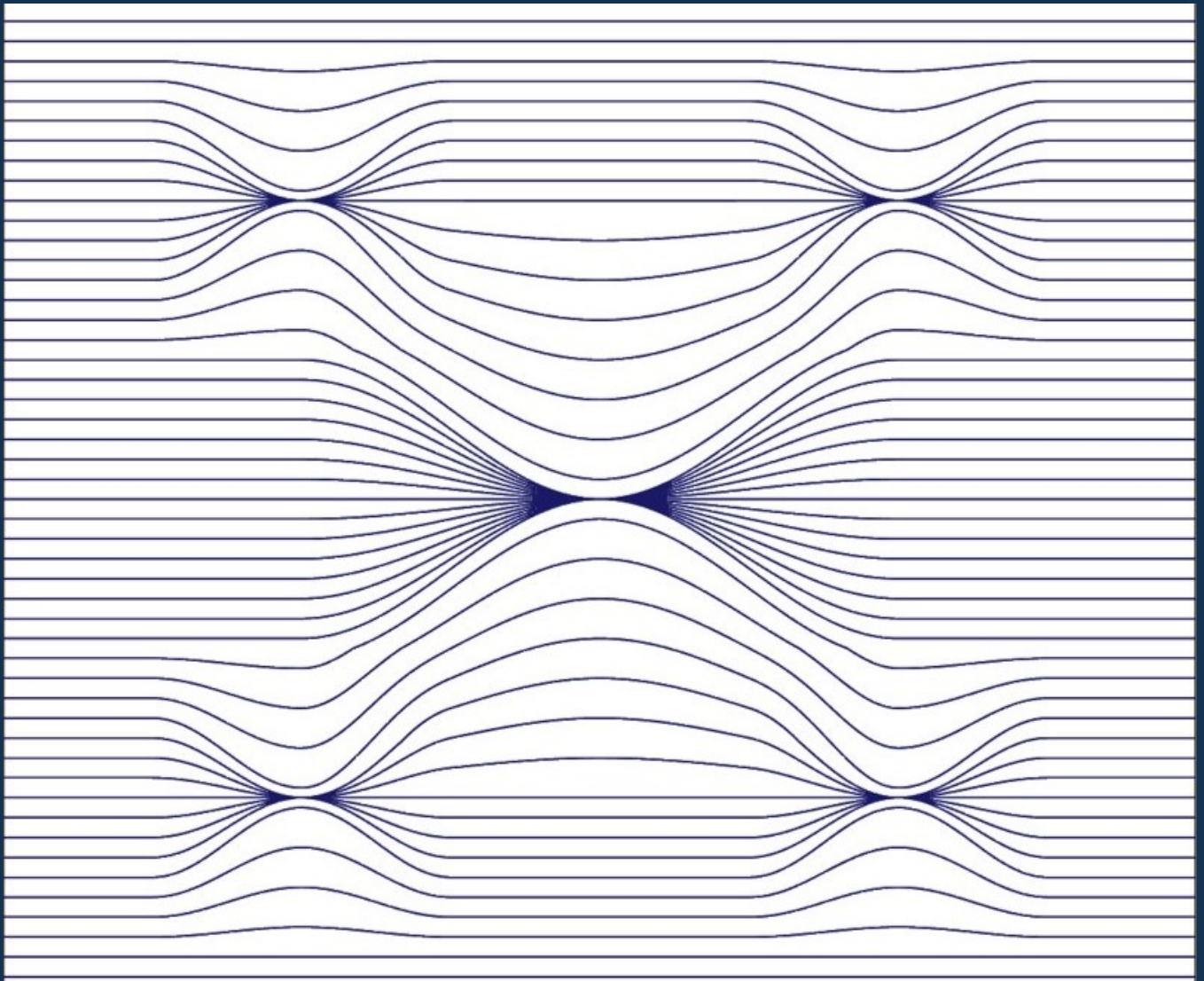




LONDON
MATHEMATICAL
SOCIETY
EST. 1865

NEWSLETTER

Issue: 514 - February 2025



CAYLEY GRAPH
AND CAYLEY
HASH FUNCTIONS

INTERFERENCE
PROTECTION
CRITERIA

NOTES OF
A NUMERICAL
ANALYST

EDITOR-IN-CHIEF

Alina Vdovina (City College of New York, CUNY)
newsletter.editor@lms.ac.uk

EDITORIAL BOARD

David Chillingworth (University of Southampton)
Jessica Enright (University of Glasgow)
Cathy Hobbs (University of Bristol)
Stephen Huggett (University of Plymouth)
Rosanna Laking (University of Verona)
Thomas Kempton (University of Manchester)
Robb McDonald (University College London)
Niall MacKay (University of York)
Yuri Santos Rego (University of Lincoln)
Mike Whittaker (University of Glasgow)
Andrew Wilson (University of Glasgow)

CORRESPONDENTS AND STAFF

News Editors: Cathy Hobbs, David Chillingworth
Mathematics News Flash Guest Editor:
Jonathan Fraser (University of St Andrews)
Typesetting: Jonathan Webley
Printing: Holbrooks Printers Ltd

EDITORIAL OFFICE

London Mathematical Society
De Morgan House
57–58 Russell Square
London WC1B 4HS
newsletter@lms.ac.uk

Charity registration number: 252660

COVER IMAGE

Credit: Nick Trefethen. See 'Notes of a Numerical Analyst'.

Do you have an image of mathematical interest that may be included on the front cover of a future issue? Email images@lms.ac.uk for details.

COPYRIGHT NOTICE

News items and notices in the Newsletter may be freely used elsewhere unless otherwise stated, although attribution is requested when reproducing whole articles. Contributions to the Newsletter are made under a non-exclusive licence; please contact the author or photographer for the rights to reproduce. The LMS cannot accept responsibility for the accuracy of information in the Newsletter. Views expressed do not necessarily represent the views or policy of the Editorial Team or London Mathematical Society.

ISSN: 2516-3841 (Print)
ISSN: 2516-385X (Online)
DOI: 10.1112/NLMS

NEWSLETTER WEBSITE

The Newsletter is freely available electronically at lms.ac.uk/publications/lms-newsletter.

MEMBERSHIP

Joining the LMS is a straightforward process. For membership details see lms.ac.uk/membership.

SUBMISSIONS

The Newsletter welcomes submissions of feature content, including mathematical articles, career related articles, and microtheses from members and non-members. Submission guidelines and LaTeX templates can be found at lms.ac.uk/publications/submit-to-the-lms-newsletter.

Feature content should be submitted to the editor-in-chief at newsletter.editor@lms.ac.uk.

News items should be sent to newsletter@lms.ac.uk.

Notices of events should be prepared using the template at lms.ac.uk/publications/lms-newsletter and sent to calendar@lms.ac.uk.

For advertising rates and guidelines see lms.ac.uk/publications/advertise-in-the-lms-newsletter.

CONTENTS

NEWS	The latest from the LMS and elsewhere	4
LMS BUSINESS	Reports from the LMS	12
FEATURES	Mathematical Notes on Interference Protection Criteria for the Fixed Satellite Service	22
	Girth of the Cayley Graph and Cayley Hash Functions	27
	AI Tools: The Facts of Licensing	31
	Notes of a Numerical Analyst	35
	Mathematics News Flash	36
OBITUARIES	In memoriam	37
EVENTS	Latest announcements	38
CALENDAR	All forthcoming events	40

LMS NEWS

Teaching and Scholarship Staff in HE Mathematics

On behalf of the London Mathematical Society's Education Committee and the Institute of Mathematics and its Applications' Higher Education Committee, in April 2024 we conducted a survey via the Heads of Departments of Mathematical Sciences network to understand the distribution of colleagues on teaching and scholarship (T&S) contracts in mathematical sciences departments. This was intended as an initial scoping exercise, with further work to follow on analysing routes to promotion for T&S colleagues.

T&S academic roles are becoming increasingly common in UK universities, in contrast to the more traditional teaching and research academic roles. In many institutions, the T&S pathway is a progression route that sits alongside teaching and research and other academic pathways on equivalent salary scales.

The reasons behind the emergence of T&S roles differ across the sector, with the rules of past Research Excellence Frameworks on staff eligibility playing a part. Furthermore, in the current highly competitive recruitment landscape, institutions recognise the need to prioritise high-quality teaching and learning. Many mathematical sciences departments now recruit into T&S roles mathematicians whose expertise is in teaching, learning and scholarship, with routes to promotion becoming increasingly available. However, expectations of what activities count as valid contributions towards promotion on a T&S pathway and what excellence looks like within T&S seem to vary considerably across UK institutions.

The Complete University Guide¹ lists 65 providers of BSc mathematics degrees, and of those, we received responses from 38 departments. Within this sample, information on about 1,801 staff was obtained, 243 of whom were on a T&S pathway (about 13.5%). Note that the sample also included 16 out of 24 Russell Group universities. Out of the 1,150 Russell Group staff, 130 were on a T&S pathway (about 11.3%). At this point, it should be emphasised that the figures are based on a sample, and it is, therefore, not clear how accurately this represents the sector as whole. Still, it does indicate that a considerable number

of colleagues are now working on T&S pathways in mathematical sciences departments.

It would also be interesting to consider whether the figure of 13.5% obtained from the survey is indicative of the UK higher education sector or of other disciplines. In this context, note that in economics, for example, a research study estimated that the proportion of staff on teaching-focused routes was 31% (aeaweb.org/conference/2024/program/1459).

The overall number of T&S staff in the mathematical sciences indicates that actions aimed at supporting these colleagues and their work on improving learning and teaching in their departments should have considerable value, both for members and in support of the pipeline of future mathematical sciences graduates. Such activities could include career-development activities and networking for staff alongside existing schemes (such as the Teaching and Learning Workshops in HE) that are sponsored by the LMS, IMA and the Royal Statistical Society. Further work to consider promotion criteria for T&S colleagues and how those are interpreted in mathematical sciences departments will help to inform future career development activities.

Catherine Hobbs
Paola Iannone
Mary McAlinden

The Society's New Data Access Policy

In response to open science mandates from funders and institutions, the Society has developed a policy on data access for its journals. The open science movement includes the open access publication of journal articles and aims to make all outcomes of research, such as code, samples, data sets, etc. as well as the published papers, open and accessible.

The Society acknowledges that many mathematical research articles do not generate research data in the usual sense. However, data are defined very broadly within the open science movement; the concept encompasses any evidence or resources that would be necessary for others to fully evaluate the basis for a

¹thecompleteuniversityguide.co.uk/league-tables/rankings/Mathematics, last accessed on 2 September 2024

paper's results and to verify or reproduce the work. This includes but is not limited to the code, software, algorithms, and raw or processed data. Research data that are not required to verify or replicate the results reported in articles (including notes from the development of the research) are not covered by this definition.

Many funding bodies now expect research data arising from their funding to be made as open as possible and only as restricted as necessary. Additionally, funders increasingly require the inclusion of data access statements (sometimes called data availability statements) within journal articles. Data access statements inform the reader if and how any data arising from the research can be accessed. Some funders (including UKRI) require the inclusion of data access statements even if no data were generated in the work.

The Society's new data access policy is designed to help mathematicians meet their funder mandates, and it recommends that researchers check the terms of their funding carefully. The Society encourages, but does not require, the open sharing of data. For those mathematicians who have data and wish to share it, the policy provides guidance on how they may do this. The Society also encourages, but does not require, the inclusion of data access statements in articles. Additionally, the policy includes template data access statements for different scenarios.

The new policy may be accessed online at lms.ac.uk/publications/policies/DataAccessPolicy.

Simon Buckmaster
Head of Academic Publications

Introducing *Levelling Up: Maths for Engineering*

The Institute of Mathematics and its Applications (IMA), in partnership with the Royal Academy of Engineering and *Levelling Up: STEM*, has launched a new initiative: *Levelling Up: Maths for Engineering*. This innovative scheme builds upon the successful *Levelling Up: Maths* scheme and incorporates new engineering-focused modules to provide a comprehensive learning experience for A level maths students considering engineering as a career. The scheme forms part of the wider *Levelling-Up STEM* initiative, which also includes schemes in physics, chemistry and computer science (levellingupstem.co.uk).

The Society is pleased to have been able to pilot and launch *Levelling Up: Maths* over the past 3 years and aims to maintain its endorsement of the *Levelling Up: Maths* scheme by providing access to academic materials and the Moodle platform. The IMA will assume administrative responsibilities for the *Levelling Up: Maths* scheme to ensure its continued growth and development.

Jennifer Gunn
Head of Society Business

Forthcoming LMS Events

The following events will take place in the forthcoming months:

LMS/BCS-FACS Seminar 2025, 15 January, online via Zoom (tinyurl.com/4zr88uup)

LMS South West and South Wales Regional Meeting 2025, 14 May, Cardiff University (tinyurl.com/2rx8924b)

A full listing of upcoming LMS events can be found on page 40 and at lms.ac.uk/events/calendar.

Support Mathematics Through Your Membership

At the heart of the LMS lies a commitment to fostering a thriving mathematical community. Our charity achieves this in many ways. We engage with our members at conferences and events and create valuable networking opportunities for career development. We actively work to make mathematics an inclusive field by funding initiatives that address under-representation and aim to create equal opportunities for all in mathematics. The LMS also provides grants and funding to support mathematical research and collaboration, enabling groundbreaking discoveries and fostering global partnerships. Through prizes, awards and lectures, our community celebrates mathematical excellence and inspires continued innovation in the field.

Why Support the LMS?

Because together, we can achieve more for mathematics than we can alone. By combining our individual resources and talents, we create a powerful force for championing mathematics and making a lasting difference.

How Can You Help?

Whether you are a mathematician, a mathematics enthusiast or simply someone who believes in the power of education and innovation, there are many ways in which you can support the LMS:

- **Become a member or renew your membership**

Join a vibrant international community of mathematicians and enjoy the benefits of networking, resources and collaboration. Visit our website to learn more about how to apply (lms.ac.uk/membership/how-join) or to renew your subscription (lms.ac.uk/membership/paying-your-subscription).

- **Publish with the LMS**

Share your academic work with our international community by publishing in LMS journals (lms.ac.uk/publications/submit-paper). Every article published in our journals directly generates more funding for the Society, and 100% of these funds are reinvested in mathematics.

- **Donate to the Society**

You can donate online (tinyurl.com/ywctzpxc), set up regular donations (tinyurl.com/4kbkzve3) or leave a legacy (tinyurl.com/ykvu2pje). Whether you wish to make a one-off contribution or become a regular donor, your support can go even further at no additional cost to you. If you are a UK taxpayer, the Society can increase the value of every donation you make by reclaiming an extra 25p for each £1 of your donation from the tax you paid.

Together, we shape the future of mathematics

Your support enables us to inspire the next generation of mathematicians, fund research and ensure mathematics continues to flourish by addressing the challenges of the modern world. By becoming part of the LMS community, you will make a meaningful impact.

Valeriya Kolesnykova
Fellowships, Membership and Accounts Assistant

OTHER NEWS

Plus Magazine Reports Back from *ECM 2024!*

Last summer, *Plus Magazine*, with the generous support of the LMS, had the exciting opportunity to attend the *9th European Congress of Mathematics (ECM)* in Seville, Spain. Over 1,300 mathematicians from all over the world gathered to share the latest breakthroughs and exciting developments in the field.

The Plus team sat down with several esteemed mathematicians, including EMS prize-winners and invited speakers, for a series of exclusive interviews. These interviews are now available as podcasts on the Plus website, offering you a chance to

- **Play a game of mathematical billiards with Giovanni Forni:** Explore the fascinating intersection of randomness and determinism in this game-themed interview.
- **Delve into the captivating world of symmetry with Jessica Fintzen:** Discover how mathemati-

cians are using symmetry to unlock new insights across various disciplines.

- **Uncover the surprising maths behind everyday networks with Richard Montgomery:** Learn how network theory helps us understand everything from traffic flow to social media connections.

Head over to plus.maths.org/content/ecm-2024 to explore all the *Plus Magazine* podcasts from *ECM 2024*.

Katherine Wright
Communications and Policy Manager

Heidelberg Laureate Forum: Calling all Young Mathematicians!

Deadline: 11 February 2025 (11.59pm CET/UTC+1)

The Heidelberg Laureate Forum (HLF) is offering 200 exclusive spots for young researchers in maths and computer science from all over the world to attend

their annual networking conference in Heidelberg, Germany, over 14–19 September 2025.

This event offers the unique opportunity to interact with laureates of the most prestigious prizes in mathematics and computer science. Traditionally, it is attended by the recipients of the Abel Prize, the ACM A.M. Turing Award, the ACM Prize in Comput-

ing, the Fields Medal, the IMU Abacus Medal and the Nevanlinna Prize.

Undergraduate, graduate and postdoctoral students are encouraged to apply. Read more and apply at application.heidelberg-laureate-forum.org.

Katherine Wright
Communications and Policy Manager

MATHEMATICS POLICY DIGEST

Maths Week Parliamentary Expo: A Celebration of Mathematical Sciences

Held during *Maths Week England 2024*, the *Maths Week Parliamentary Expo* brought together a diverse group of leaders from academia, education, industry and politics. This event showcased the vital role of mathematical sciences in driving research, innovation and economic prosperity.

Organised by the Campaign for the Mathematical Sciences (CaMS), the expo featured interactive displays highlighting the latest advances in the UK in the mathematical sciences. Attendees had the opportunity to engage with leading institutions like the LMS, Heilbronn Institute, UK Knowledge Exchange Hub for Mathematical Sciences and Maths for Deep Learning.

Hosted by Chi Onwurah MP, the event featured insightful speeches from prominent figures including Professor Catherine Hobbs, Professor Aoife Hunt MBE, Dr Ben Spencer (Shadow Minister of State for Science, Innovation and Technology) and Dr Ian Solom (Liberal Democrat Spokesperson for Universities and Skills).

The expo provided a valuable platform for directly engaging with newly elected MPs and strengthening existing relationships with CaMS supporters.

TIMSS 2023: A Snapshot of English Students' Maths Performance

Initial findings from the *Trends in International Mathematics and Science Study (TIMSS) 2023* have been released by the Department for Education. This international benchmark assesses the mathematical and scientific knowledge of students in Years 5 and 9.

The report provides a detailed analysis of England's performance compared to other participating countries. To delve deeper into the findings, visit timssandpirls.bc.edu/timss2023.

Digest prepared by Katherine Wright
Communications and Policy Manager

Note: Items included in the Mathematics Policy Digest are not necessarily endorsed by the Editorial Board or the LMS.

OPPORTUNITIES

LMS–Sheffield Mathematical Symposia 2026

Call for Proposals

Submission deadline: 3 March 2025

The London Mathematical Society (LMS) is pleased to announce its call for proposals for the *LMS–Sheffield Mathematical Symposium* to be held at the University of Sheffield in 2026. Starting life in 1974 as the *LMS–Durham Symposia* and then continuing as the *LMS–Bath Symposia* after 2021, the *LMS–Sheffield Mathematical Symposia* will be held at the University of Sheffield between 2026 and 2030. The symposia are an established and recognised series of international research meetings that provide an excellent opportunity to explore an area of research in depth, to learn of new developments and to instigate links between different branches.

To support the symposium, there is core funding of approximately £16k to £20k, which will be provided by the University of Sheffield’s School of Mathematical and Physical Sciences, the LMS, the International Centre for Mathematical Sciences (ICMS) and the Isaac Newton Institute (INI). Pending the outcome of a current EPSRC grant application, the core funding may increase. In addition to the core support, organisers are encouraged to seek further funding from other funding bodies, industry partnerships or their local institutions.

Proposals are now invited for meetings. They should be held during July or August and be open to all. A significant proportion of the participants are expected to come from the UK. The organisers have the option to complement a symposium with a summer school to prepare young researchers, such as PhD students, or a ‘research incubator’, where problems related to the topic of the conference are studied in smaller groups. Where appropriate, prospective organisers should consider the possibility of an ‘industry day’.

Prospective organisers should send a formal proposal to the LMS Grants Team (grants@lms.ac.uk) by 3 March 2025. Proposals will be approved by the LMS Research Grants Committee after careful consideration of referees’ reports.

Your Proposal

Your proposal should include the following and be submitted in PDF format:

- A list of organisers. Indicate, if any, those who are female or an early career researcher.
- A list of key invited participants. Indicate those who have confirmed their participation. Break down the number of attendees by gender, career stage and geographical location (within the UK and the world). Explain your efforts to improve the diversity of the participants. Please note the advice on diversity (lms.ac.uk/women/speaker-diversity-conferences-seminars).
- A detailed scientific case for the symposium, including its objectives, novelty and timeliness. Explain how the meeting will impact UK mathematics.
- A list of any other recent or upcoming meetings on similar topics and discuss how this meeting differs from them.
- Details of any additional support from other funding bodies or proposed avenues of available funding that the organisers are planning to apply for.
- Details of any plans for a summer school or a research incubator.
- If applicable, details of the industry day.

For further details about the *LMS Mathematical Symposia*, please visit the Society’s website (lms.ac.uk/events/mathematical-symposia).

Before submitting their proposal, organisers are welcome to discuss their ideas informally with the Chair of the Research Grants Committee, Professor Amanda Turner (grants@lms.ac.uk).

Lucy Covington
Grants Administrator

LMS Grant Schemes

Research Grants

Next application deadline: 15 May 2025

Applicants for LMS grants should be mathematicians based in the UK, the Isle of Man or the Channel Islands. For grants to support conferences or workshops, the event must be held in the UK, the Isle of Man or the Channel Islands.

Scheme 1 — Conference and Workshop Grant

Grants of up to £5,500 are available to provide partial support for conferences and workshops. This includes travel, accommodation and subsistence expenses for principal speakers, UK-based research students, participants from Scheme 5 countries and the caring costs for attendees who have dependants.

Scheme 2 — Visitors to the UK Grant

Grants of up to £1,500 are available to provide partial support for a visitor who will give lectures in at least three separate institutions. Awards are made to the host towards the travel, accommodation and subsistence costs of the visitor. Potential applicants should note that it is expected that the host institutions will contribute to the costs of the visitor. In addition, the Society can offer a further amount (of up to £200) to cover the caring costs for those who have dependants.

Scheme 4 — Research in Pairs Grant

For those mathematicians inviting a collaborator, grants of up to £1,200 are available to support a visit for collaborative research, either by the grant holder to another institution abroad or by a named mathematician from abroad to the home base of the grant holder. For those mathematicians collaborating with another UK-based mathematician, grants of up to £600 are available to support a visit for collaborative research, either by the grant holder to another institution or by a named mathematician to the home base of the grant holder. In addition, the Society can offer a further amount (of up to £200) to cover the caring costs for those who have dependants.

Scheme 4 — Research Reboot Grant

Grants of up to £500 for accommodation, subsistence and travel plus an additional £500 for caring

costs are available to assist UK mathematicians who may have found themselves with very little time for research due to illness, caring responsibilities, increased teaching or administrative loads, or other factors. This scheme offers funding so that they can leave their usual environment to focus entirely on research for a period from two days to a week.

Scheme 5 — Collaborations with Developing Countries

For mathematicians inviting a collaborator to the UK, grants of up to £3,000 are available to support a visit for collaborative research by a named mathematician from a country in which mathematics could be considered to be in a disadvantaged position to the home base of the grant holder. For mathematicians going to their collaborator's institution, grants of up to £2,000 are available to support a visit for collaborative research by the grant holder to a country in which mathematics could be considered to be in a disadvantaged position. Applicants will be expected to explain in their application why the proposed country fits the circumstances considered eligible for Scheme 5 funding. In addition, the Society can offer a further amount (of up to £200) to cover the caring costs for those who have dependants.

Contact the Grants Team if you are unsure whether the proposed country is eligible or check the IMU's Commission for Developing Countries definition of developing countries (tinyurl.com/y9dw364o).

Mathematics in Africa Grant

Grants of up to £2,000 are available to provide partial support for mathematical activities based in Africa, including attending a conference or workshop, organising a conference or workshop, or undertaking a mathematical research collaboration.

Lucy Covington
Grants Administrator

2025 Grace Chisholm Young Fellowship

Call for Applications

Application deadline: 28 February 2025

Are you facing a career break due to family commitments, relocation or other similar circumstances? The LMS offers the annual Grace Chisholm Young Fellowship to support mathematicians in restarting their careers.

This fellowship provides a £1,500 personal research support fund to help you focus on your research without financial worry. The fellowship holder must be based in a UK mathematics department at a university or research institute. The host institute will receive a contribution of £500 from the LMS and be expected to provide an email address, use of library and IT facilities for correspondence, and access to research literature.

To be eligible, you must have at least submitted your PhD thesis. Applications are not gender restricted.

For further details and to download an application form, go to lms.ac.uk/grants/grace-chisholm-young-fellowships.

Submit your application to Kieran O'Connor, Events Co-ordinator, via womenanddiversity@lms.ac.uk.

Katherine Wright
Communications and Policy Manager

David Crighton Medal 2025

Call for Nominations

Nominations are invited for the LMS/IMA David Crighton Medal in 2025. The medal is awarded to an eminent mathematician for services both to mathematics and to the mathematical community.

The David Crighton Medal was established by the Councils of the LMS and the Institute for Mathematics and its Applications (IMA) to pay tribute to the memory of Professor David George Crighton FRS. The prize-winner must be resident in the UK on 1 January of the year of the award. The prize-winner will receive a silver gilt medal, which will be presented at a joint meeting of the LMS and the IMA, and will be invited to give a lecture.

Previous winners of the David Crighton Medal are Professor Alison Etheridge (2023), Professor Caroline Series (2021), Professor Ken Brown (2019), Professor I. David Abrahams (2017), Professor Frank Kelly (2015), Professor Arieh Iserles, Dr Peter Neumann (2012), Professor Keith Moffatt (2009), Professor Sir Christopher Zeeman (2006) and Professor Sir John Ball (2003).

Read more about the David Crighton Medal and download a nomination form at lms.ac.uk/prizes/david-crighton-medal. Please send any queries to Katherine Wright, Secretary to the David Crighton Medal Panel: prizes@lms.ac.uk. The completed nomination form should be sent to prizes@lms.ac.uk by 28 February 2025.

Katherine Wright
Communications and Policy Manager

VISITS

Visit of JeongHyeong Park

Professor JeongHyeong Park from Sungkyunkwan University (SKKU) in the Republic of Korea will visit the UK from 9 to 27 February 2025. Professor Park will be accommodated in Durham from 9 to 23 February and in Oxford from 23 to 27 February. The visit will enable her to collaborate with Norbert Peyerimhoff, her host at Durham University, on harmonic and asymptotic harmonic manifolds and with Stuart Hall (Newcastle University) and Andrew Dancer (University of Oxford) on Einstein manifolds and generalisations. During her visit, Professor Park will give three talks:

- *Leeds Geometry Seminar*, 12 February 2025
- *Durham Geometry/Topology Seminar*, 13 February 2025
- *Newcastle Algebra and Geometry Seminar*, 18 February 2025

For further details contact Norbert Peyerimhoff at Durham University.

The visit is supported by an LMS Scheme 2 grant.

Visit of Sat Gupta

Professor Sat Gupta will be visiting the Department of Mathematical Sciences, Durham University, from 16 to 27 March 2025. Professor Gupta is Professor of Statistics at the University of North Carolina, Greensboro. In recent years, his research has mostly been on the theory of randomised response methods, which are used to elicit possibly sensitive information from study participants. During his visit, Professor Gupta will give lectures at:

- Brunel University of London, 17 March 2025 (contact Keming Yu: keming.yu@brunel.ac.uk)
- City University London, 18 March 2025 (contact Mark Broom: mark.broom.1@city.ac.uk)
- Durham University, 26 March 2025 (contact Frank Coolen: frank.coolen@durham.ac.uk)

For further details, contact Frank Coolen (frank.coolen@durham.ac.uk).

The visit is supported by an LMS Scheme 2 grant.

Visit of Glenn Stevens

Professor Glenn Stevens will be visiting the UK from 1 February to 29 March 2024. Professor Stevens is a number theorist from Boston University. He has made major contributions to the theory of p -adic modular forms. During his visit Professor Stevens will give lectures at:

- University of Bristol, 12 February 2025
- Oxford University, 14 March 2025
- Kings College, London, 21 March 2025

For further details contact Owen Patashnick (owen.patashnick@kcl.ac.uk). The visit is supported by an LMS Scheme 2 grant, Merton College, Oxford, Kings College Research visiting programme, the University of Bristol and the Heilbronn Institute for Mathematical Research. We thank them all for their support.

Visit of Roozbeh Hazrat

Professor Roozbeh Hazrat will be visiting the Mathematical Sciences Research Centre, Queen's University Belfast, from 1 to 9 May 2025. Professor Hazrat is a member of the School of Computer, Data and Mathematical Sciences of Western Sydney University. His main research interests are graded algebra, Leavitt path algebras and algebraic K -theory. During his visit, Professor Hazrat will give lectures at:

- Queen's University Belfast, 8 May 2025 (contact Thomas Huettemann: t.huettemann@qub.ac.uk)
- University of Edinburgh, 12 May 2025 (contact Sue Sierra: S.Sierra@ed.ac.uk)
- University of Cambridge, 14 May 2025 (contact Adam Jones: aj718@cam.ac.uk)

For further details, contact Thomas Huettemann (t.huettemann@qub.ac.uk).

The visit is supported by an LMS Scheme 2 grant.

LMS Elections 2024/25

We are pleased to report the results of the LMS Elections to Council and Nominating Committee in 2024. These were announced at the Annual General Meeting on 22 November 2024 following a vote by the LMS membership.

Officers of Council

President-Elect:	Mark Chaplain
Vice-President:	Iain Gordon
Vice-President:	Catherine Hobbs
General Secretary:	David Barnes
Treasurer:	Simon Salamon
Publications Secretary:	Niall MacKay
International Secretary:	Minhyong Kim
Education Secretary:	Mary McAlinden

New Members-at-Large of Council (2-year terms)

Member-at-Large:	Peter Ashwin
Member-at-Large:	Lassina Dembélé
Member-at-Large:	Clare Dunning

Member-at-Large:	Jason Lotay
Member-at-Large:	Amanda Turner
Member-at-Large:	Sarah Whitehouse

Nominating Committee

The following members were elected to the LMS Nominating Committee: David Abrahams (three-year term), Carola-Bibiane Schönlieb (three-year term) and Agelos Georgakopoulos (one-year term). Continuing members of the Nominating Committee are Helen Wilson (Chair), Karin Baur, Laura Ciobanu and Victoria Gould.

We congratulate the newly elected members of the Council of Trustees and the LMS Nominating Committee and look forward to their contributions to the Society.

Katherine Wright
Communications and Policy Manager



HEILBRONN INTERNATIONAL VISITORS SCHEME - Call for proposals

The Heilbronn Institute for Mathematical Research offers UK based academics the opportunity to apply for funding to host leading international academics for short mathematical research visits to the UK. Proposals are invited for visitors whose research areas include, but are not restricted to: Algebra, Algebraic Geometry, Combinatorics, Data Science, Number Theory, Probability, and Quantum Information.

Applications can be submitted at any time and will be considered by the Chair of the Heilbronn Institute on a rolling basis, with a maximum award of £10,000 to support visits of up to 3 weeks.

The purpose of the scheme is to raise the profile and enrich the research environment of the host department. Visits funded previously under this scheme have resulted in research collaborations, well attended talks, seminars, and colloquia.

This scheme is solely funded through the UKRI/EPSCRC 'Additional Funding Programme for Mathematical Sciences'.

Two A4 page proposals should be sent to: heilbronn-manager@bristol.ac.uk For further particulars and additional information, please [visit our webpages](#).

Members of Council 2024/2025



Jens Marklof
President



Cathy Hobbs
Vice-President



Iain Gordon
Vice-President



Simon Salamon
Treasurer



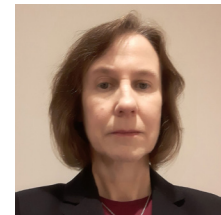
David Barnes
General Secretary



Minhyong Kim
International Secretary



Niall MacKay
Publications Secretary



Mary McAlinden
Education Secretary



Sara Lombardo
Member-at-Large (Women
and Diversity)



Peter Ashwin
Member-at-Large



Andrew Brooke-Taylor
Member-at-Large



Elaine Crooks
Member-at-Large



Lassina Dembélé
Member-at-Large



Clare Dunning
Member-at-Large



Jessica Enright
Member-at-Large



Jason Lotay
Member-at-Large



Rachel Newton
Member-at-Large



Gregory Sankaran
Member-at-Large



Amanda Turner
Member-at-Large



Sarah Whitehouse
Member-at-Large

Retiring Members of Council

Robb McDonald has stepped down as General Secretary after a four-year term, having been first elected in 2020. Robb was elected at the time of the pandemic, when we had to move to remote working overnight. He expertly helped to steer us through this time and ensured that we were compliant and that our governance remained effective. Robb put in place the changes to the governance agreed in 2019. Moreover, he has always been available for advice and has provided a clear and pragmatic way forward with the interests of the Society at its core.

Chris Parker has stepped down as Programme Secretary after six years as an Officer, having been first elected in 2018. Chris served as Chair of the Early Career Research (ECR) Committee from 2017 to 2023 and before that was Chair of the Research Meetings Committee from 2016 to 2017. As Chair of the ECR Committee, he oversaw activities such as the Undergraduate Summer Schools and the LMS Research Schools as well as grant schemes like the Undergraduate Research Bursaries and the Cecil King Travel Scholarships. In recent years, he has spearheaded a workshop series with ICMS focused on encouraging undergraduates in the mathematical sciences to consider undertaking mathematics research degrees.

Anne Taormina, Member-at-Large, completed six years as a Member of Council. Anne was the Member-at-Large for Membership and was responsible for updating the Council on issues relating to the LMS membership and LMS representatives. She also served on a number of LMS committees, including the Committee for Women and Diversity in Mathematics and the Personnel Committee.

We thank Robb, Chris and Anne for their valuable service to the Society.

Katherine Wright
Communications and Policy Manager

Professor Peter Tankov Awarded the 2024 Louis Bachelier Prize

The prestigious Louis Bachelier Prize was awarded to Professor Peter Tankov (Institut Polytechnique de Paris) on 16 October 2024 during a ceremony held at the Natixis Group headquarters in Paris. The event brought together leading figures in the field of mathematical finance to celebrate Professor Tankov's significant contributions.



From left to right: Iain Gordon, Peter Tankov, Jean Cheval and Michel Crouhy

Professor Tankov received recognition for his groundbreaking work in green finance, climate risk and sustainable finance. This research tackles crucial issues facing the financial world today. Additionally, the award acknowledges his broader expertise in Lévy processes, which play a vital role in financial modelling, and his development of advanced computational methods used in the financial sector. Professor Tankov's research exemplifies the power of rigorous mathematical and statistical modelling in risk management and market regulation and demonstrates the profound impact of mathematics on the financial world.

The Louis Bachelier Prize, awarded biennially, includes a €20,000 cash prize and a dedicated £5,000 for organising a European scientific workshop on the winner's research area. Learn more about the prize at lms.ac.uk/prizes/louisbachelierprize.

Katherine Wright
Communications and Policy Manager

A Report on the *Black Heroes of Mathematics Conference 2024*

JORDAN MARAJH

The *Black Heroes of Mathematics Conference 2024* celebrated the contributions of Black mathematicians and highlighted their pioneering research, leadership and efforts to promote diversity. The conference fostered inspiration, education and empowerment for the next generation of mathematicians through presentations and discussions. Moreover, 2024 marked the first in-person edition of the conference.

Introduction

The *Black Heroes of Mathematics Conference 2024* (BHoM24) was the first in-person edition of the conference, which started in 2020 and has been run virtually until now. It was also the first edition, but certainly not the last, that I have attended personally. It was delightful to see so many prominent figures in mathematics joining us, both online and in the room at De Morgan House. However, one of the best parts was seeing other students and teachers from the wider mathematical community in attendance.

The conference ran over two days, Wednesday, 2 October, and Thursday, 3 October, and was chaired expertly by Professor Nira Chamberlain OBE. There were four talks on each day with a following panel session. During the breaks, the collaborative spirit among attendees drew us together to discuss the speakers' research, their paths to where they are now and their continued work in outreach for Black mathematicians.

This remainder of this article aims to summarise key aspects of each talk and highlight some of the discussions that took place in the panel sessions. I will conclude with what are my key takeaways from BHoM24, and hopefully, this applies to many others in the wider mathematical community.

Modelling and Mapping Malaria Risk in Ghana — Justice K. Aheto

Professor Justice K. Aheto presented his innovative work on the spatial-temporal modelling of malaria risk in Ghana using real data collected from 2016 to 2021 spanning the 261 districts in Ghana. Aheto's models predict malaria hotspots, which supports targeted, efficient interventions. By incorporating socio-

economic and environmental factors, such as population density and water proximity, his research addresses the challenges of tackling malaria in resource-limited settings. His work underscores the crucial role that statistical modelling plays in public health and enables policymakers to make informed decisions that could save lives and improve health outcomes. Aheto demonstrated that a simple and effective web-based implementation of this tool could serve as a valuable framework for malaria control and could be applied globally to other infectious diseases.

Catch up on anything you missed!

All talks were recorded by the LMS and can be found on YouTube [1]. There are many details that I certainly could not convey as well as the original speaker, so I urge the reader to watch or listen to any of the great talks given.

On the other hand, the panel sessions were not recorded. This had the advantage of fostering open, stress-free discussions surrounding many topics affecting the Black community. There were many questions that the panellists gave their views on, and these quite frequently prompted the audience to chime in with how to remedy issues facing the community.

Modelling Low-Energy Electron Emission — Imoleayomide Ajayi

Imoleayomide Ajayi shared her research on modelling low-energy electron emissions using truncated normal generalised linear models (TN-GLM). Her interdisciplinary approach blends statistical methods

with material sciences and focuses on applications in microplastic imaging and scanning electron microscopy (SEM). Ajayi highlighted the potential of advanced statistical methods, like TN-GLM, to solve real-world scientific problems and encouraged the development of future interdisciplinary collaboration to address challenges at the intersection of mathematics, science and technology. Her personal journey and resilience in pursuing a career in STEM added a compelling narrative to the discussion. Her maths journey started in Ghana, and today, she is a PhD student at Loughborough University.



A photo from the BHoM24 conference hosted at De Morgan House. From left to right: Kabiru Abubakari, Professor Nira Chamberlain OBE and Jordan Marajh.

Mathematical Insights into Social Dynamics — Nathalie Ayi

Nathalie Ayi's talk offered a deep dive into mathematical models of social dynamics, particularly in opinion formation, polarisation and consensus in large populations. She discussed how agent-based models, mean-field theories and graph-limit approaches are used to analyse social behaviour, with applications in understanding collective decision-making and the effects of social networks. Ayi highlighted the potential of using kinetic theory and the associated PDEs to model social interactions and how these mathematical frameworks could contribute to building an understanding of societal challenges, like political polarisation and misinformation. Her work provides valuable insights into how mathematical theory can help address contemporary issues in social dynamics and policy development.

Scoping Maths Anxiety in Our Community — Flavia H. Santos

Dr Flavia Santos discussed the pervasive issue of mathematics anxiety. She explored its physiological, cognitive and emotional dimensions. Drawing from psychological and neuroscientific research, she explained how maths anxiety can negatively impact both academic performance and long-term student confidence. Santos emphasised that this issue is not merely an individual challenge but one deeply tied to cultural and environmental factors and is often exacerbated by a lack of supportive learning environments. By advocating for evidence-based strategies to reduce maths anxiety [4], her talk underscored the need for inclusive, trauma-informed approaches in education that can help students build resilience and develop healthier relationships with mathematics. Overall, she gave a very balanced talk at De Morgan House.

Dispersed Regression Models for Dispersed Count Data — Kimberly Sellers

Professor Kimberly Sellers presented a comprehensive discussion on advanced statistical methods for modelling over- and under-dispersed count data, with a particular focus on the Conway–Maxwell–Poisson model. Sellers illustrated the flexibility of the model in addressing real-world issues in which traditional regression models fail to account for the variability seen in empirical data. Her work demonstrated the applicability of these models in diverse fields such as epidemiology and environmental studies, where data often show more or less dispersion than expected. Sellers emphasised the need for nuanced statistical tools to improve predictions and provide more accurate insights into complex phenomena, even one as simple as traffic modelling, which she used as one of her motivating examples.

Representation in Mathematics Education — Teresa Senyah

Teresa Senyah focused on the vital importance of representation in mathematics education, arguing that fostering positive relationships with mathematics from an early age is key to helping students, particularly those from under-represented backgrounds, engage with the subject. Drawing from her teaching

experiences, she illustrated how creating an inclusive and supportive environment where students feel seen and encouraged is essential for building confidence and success in mathematics. Senyah stressed the importance of relatable role models, student-centred learning and collaborative teaching strategies in ensuring that all students, regardless of background, see mathematics as an accessible and worthwhile pursuit. Her talk called for a shift in how mathematics is taught, and she advocated for pedagogical practices that focus on the student experience. One key point was the direct encouragement that you can give, regardless of the stage you have reached in your maths career maths, and how vital that can be to a more junior colleague or to a student's mathematical journey.

Becoming the World's Most Interesting Mathematician — Angela Tabiri

Angela Tabiri shared her inspiring journey in mathematics and underscored the importance of mentorship and resilience in pursuing STEM education, particularly for African women. Through her leadership at the African Institute for Mathematical Sciences as the academic manager for the Girls in Mathematical Sciences Program, Tabiri has mentored over 120 students. Her work emphasises the power of representation. When students see role models who reflect their own experiences, they are more likely to succeed. Tabiri's advocacy for visibility, leadership and mentorship in mathematics, particularly for African women, highlights the transformative power of education in breaking down barriers and empowering the next generation of mathematicians. She provided excellent pedagogical examples of non-commutativity in the real world, such as in language, which she applies when explaining her research to a wider audience of non-experts and which resonates with her efforts to break down barriers to entry to maths. It is no wonder that she became the World's Most Interesting Mathematician of 2024 [2].

Bob Moses and the Movement for Math Literacy as a Civil Right — Robin Wilson

Professor Robin Wilson paid tribute to Bob Moses, the founder of the Algebra Project, which has deep foundations from the civil rights movement in the USA, in the fight for maths literacy. Moses drew comparisons between the denial of maths literacy as a

modern equivalent to the historical denial of voting rights [3] and, hence, highlighted the dangers that faced Black America at the time. The Algebra Project aims to help underserved students achieve skills in algebra and frames the essentiality of algebra as raising the floor of maths literacy. Wilson emphasised that Moses's work transcended education, as he viewed maths literacy as a civil right that empowers marginalised communities. Through the Algebra Project, Moses not only advocated for educational equality but also showed how mathematics can be a transformative tool for societal justice and equity.

Panel discussions

The panels on both days included questions that were prepared ahead of time as well as those sparked by the natural flow of conversation. This created a natural environment where a conversation developed between the panellists and the general audience. Some particularly interesting questions were posed to the panel:

- What does being a Black hero of mathematics mean to you?
- What does Black History Month mean to you?
- What danger is there, if any, if Black people are not involved in the development of AI?
- Do you have any advice for young people today?
- What made you choose maths?

Many of the responses gave constructive advice on how to navigate obstacles, and some mentioned those who had inspired or encouraged them early or late in their career. A specific comment from the discussion was insightful, as it pertained to whether Black History Month would always be needed. The general consensus was that it is here to stay and was not designed to be a stopgap for celebrating Black history and doing so in just one month. Rather, it is more of an annual checkpoint to remind ourselves and future generations of the many figures driving our community, of some of the struggles we face and of how we can plan to celebrate Black history throughout the year.

An earnest discussion on AI noted that biases can be built into machine learning models without malicious intent. To remedy any potential issues, the maths

community must ensure that our youth are aware of the danger. In particular, Paulette Watson MBE has focused heavily on Black women in STEM and has given them a voice. She has written a book on her experiences [5] that contains many notable lessons, which she endeavours to teach as the founder of Academy Achievers, an organisation dedicated to closing the digital divide.

On the other hand, some questions were more light-hearted, but it was just as interesting to hear the responses to them. For instance, a question raised in the discussion was how to alleviate the stigma associated with maths, which is the Marmite of school subjects: you either love it or hate it. Similarly, the panels on both days of BHoM24 closed with a quick question: “What is your favourite film and why?” The responses were varied and informative, to name a few: *Hidden Figures*, *Back to the Future* and my personal favourite — *Star Wars: Episode III — Revenge of the Sith*. When asked to explain why, I mentioned that I always admired the calm, collected demeanour of Obi-Wan Kenobi and compared this to the patience required to do mathematics. Sometimes when I encounter roadblocks in my work, I remember the composure of old Ben Kenobi as he waited for the correct opportunity to arise (as a master of the Soresu fighting style).

Conclusion

In summary, the BHoM24 conference was an excellent occasion, and I am eagerly awaiting the next one. There were many key messages that I have taken away and have shared with other members of the wider maths community pertaining to the following:

- (1) It is never too early to encourage a budding mathematician. You may be the very reason their view of the subject changes.
- (2) Even attempting to be a role model inspires others, whatever level they have attained in their career.
- (3) Becoming masters of statistical modelling and AI will give anyone, especially the Black community, a greater voice in the years to come.

- (4) To poach Nira’s inspirational words:

You don’t need permission to be a great mathematician!

Living examples of greatness act as a catalyst to others and show them that it *is* possible to succeed.

Acknowledgements

I would like to thank Professor Nira Chamberlain OBE for the invitation to BHoM24 and for the opportunity to contribute as an invited panellist. Furthermore, I also thank all the attendees for ensuring that the conference was productive and informative for many members of the wider mathematical community.

FURTHER READING

- [1] LMS BHoM24 Speakers Playlist (videos), tinyurl.com/4rwawzne
- [2] C. Lawson-Perfect, The Big Internet Math-Off 2024, the final!, *Aperiodical*, 23 July 2024, tinyurl.com/23xhc5ud.
- [3] R.P. Moses, C.E. Cobb Jr, *Radical Equations: Math Literacy and Civil Rights*, Beacon Press, 2001.
- [4] F.H. Santos, *A Child with Maths Anxiety May Become a Parent or a Teacher with Maths Anxiety*, Geary Institute for Public Policy, 2022.
- [5] P. Watson, *She, Disrupts: A Black Woman’s Journey in the STEM and AI Industries*, Lulu Press, 2024.



Jordan Marajh

Jordan is a third-year PhD student in SMS at QMUL. His research interests are in mathematical relativity, specifically in conformal methods, differential geometry and the analysis of hyperbolic PDEs. He is rarely found without a set of rings on but occasionally takes them off when the going gets tough when playing the classic rhythm game *osu!*

REPORTS OF THE LMS

Records of Proceedings at LMS meetings Annual General Meeting and Naylor Lecture 2024

This meeting was held on Friday, 22 November 2024, at the BMA House, Tavistock Square, London, and online via Zoom. Over 100 members and guests were present, either in person or online, for all or part of the meeting.

The meeting began at 3pm with the President, Professor Jens Marklof FRS, in the Chair.

The minutes of the General Meeting, which was held on 28 June 2024, were circulated to members 21 days in advance of this meeting. Copies of those minutes were also available at the meeting, and the minutes were confirmed by those present (both in person and online).

The Vice-President, Professor Cathy Hobbs, presented a report on the Society's activities in 2023/2024 and the President invited questions. The Treasurer, Professor Simon Salamon, presented his report on the Society's finances during the 2023/2024 financial year and the President invited questions. Copies of the Trustees' Report for 2023/2024 were made available, both 21 days in advance and on the day. The President invited members to adopt the Trustees' Report for 2023/2024 by a show of hands for those in person and via a poll for those joining online. The Trustees' Report for 2023/2024 was adopted.

Forty-four new members were elected to the Society at this meeting. All members who had not yet signed the membership book were invited to do so.

The LMS Scrutineer, Professor Cho-Ho Chu, announced the results of the ballot. The following Officers and Members of the Council were elected.

President-Elect:	Professor Mark Chaplain
Vice-President:	Professor Iain Gordon
Vice-President:	Professor Catherine Hobbs
Treasurer:	Professor Simon Salamon
General Secretary:	Dr David Barnes
Publications Secretary:	Professor Niall MacKay
International Secretary:	Professor Minhyong Kim
Education Secretary:	Professor Mary McAlinden

The following were elected Members-at-Large of Council for two-year terms: Professor Peter Ashwin, Dr Lassina Dembélé, Dr Clare Dunning, Professor Jason Lotay, Professor Amanda Turner and Professor Sarah Whitehouse

The following members were elected to the LMS Nominating Committee: David Abrahams (three-year term), Carola-Bibiane Schönlieb (three-year term) and Agelos Georgakopoulos (one-year term). Continuing members of the Nominating Committee are Helen Wilson (Chair), Karin Baur, Laura Ciobanu and Victoria Gould.

The President presented the LMS Prizes for 2024:

Pólya Prize:	Professor Gui-Qiang G. Chen
Senior Berwick Prize:	Professor Christopher J Bishop
Fröhlich Prize:	Professor Emmanuel Breuillard
Anne Bennett Prize:	Dr Ana Ros Camacho
Whitehead Prizes:	Dr Sabine Bögli
	Dr Viveka Erlandsson
	Professor James Newton
	Dr Clarice Poon
	Dr Julian Sahasrabudhe
	Professor Alessandro Sisto

The winner of the Shepherd Prize, Professor Samir Siksek, was unable to collect their certificate at the meeting.

The President introduced the supporting lecture given by Professor Marco Fontelos (Madrid), who presented a lecture on *Singularities in Partial Differential Equations*.

Following the tea break, the President introduced the *Naylor Lecture 2024*. Professor Jens Eggers (Bristol University) delivered a lecture on *The Role of Singularities in Hydrodynamics*.

The President, on behalf of the meeting, thanked the following people who had served the Society over the past few years and who were now retiring from their respective positions. He acknowledged that volunteers were critical to the Society's effectiveness. He extended his gratefulness for all their hard work and time donated so generously to the Society.

Professor Robb McDonald, General Secretary (2020/2024)
 Professor Christopher Parker, Programme Secretary (2018/2024)
 Professor Anne Taormina, Member-at-Large (2018/2024)
 Professor Tara Brendle, Chair, Nominating Committee (2021/2024)

The President, on behalf of LMS Council, extended the gratitude of the Society to Susan Oakes, who retired from the Society in July 2024 after 43 years of service. Susan was the first paid member of LMS staff when she was appointed as an administrative assistant back in 1981. In recent years, she had worked on the *LMS Newsletter*, commissioning and fielding incoming contributions and ensuring the full diversity of LMS activities was communicated to the membership.

The President concluded the meeting, thanking everyone involved in organising the Annual General Meeting and the audience for their attendance. The next Society meeting is scheduled for Friday, 4 July 2025.

Following the meeting, a reception was held at BMA House, London. This was followed by the Annual Dinner, also held at BMA House, which was attended by 80 guests. To commence the Annual Dinner, President Professor Jens Marklof FRS delivered a brief speech. In his speech, he expressed gratitude to the retiring members of the Council and committee chairs as well as to Susan Oakes for her long-standing service to the Society.

Nicola Goldie
 Committee, Grants and Membership Manager

LMS Mary Cartwright Lecture 2024

This meeting was held on Wednesday, 20 November 2024, online via Zoom.

The meeting began at 2pm with the Chair of the Committee for Women and Diversity in Mathematics, Professor Sara Lombardo (Heriot-Watt University), in the Chair.

Professor Lombardo introduced the first lecture, which was given by Francesca Fedele (University of Leeds) on *Presentations of Reflection Groups, Part 1*. This was followed by Bethany Marsh (University of Leeds), who gave the *Mary Cartwright Lecture 2024* entitled *Presentations of Reflection Groups, Part 2*.

Professor Lombardo expressed her gratitude to the speakers for their outstanding lectures and extended the Society's appreciation to the organisers for hosting a successful meeting.

Nicola Goldie
Committee, Grants and Membership Manager

Sustainability Update

The LMS is committed to reducing its environmental impact through a comprehensive Corporate Environmental Policy that emphasises energy efficiency and sustainability. This policy is designed to ensure that the Society not only complies with environmental regulations but also integrates sustainable practices throughout its operations with the aim of minimising its ecological footprint.

The overarching goal is to reduce our environmental impact by prioritising energy-efficiency, conserving water, minimising waste and enhancing our communications around sustainable practices. As part of this commitment, we regularly identify cost-effective energy measures and aim to source energy sustainably, wherever possible.

In 2020, we took an important step by conducting an energy audit and establishing a programme of regular energy reviews. Through these audits, we have identified short-term objectives, which initially focused on energy-saving measures for De Morgan

House, such as minimising printing and enhancing our recycling processes to reduce waste.

Looking ahead, our medium- and long-term goals are to continue reducing energy usage and costs, to incentivise low-carbon travel through our grant schemes and to ensure that the environmental impact of new activities is accounted for. This holistic approach aims to incorporate sustainability considerations into every aspect of the Society's work.

To achieve these goals, the LMS has set up a new working group to drive forward initiatives. We are also committed to transparently reporting on our progress in our Annual Trustees' Report and sharing updates with the mathematical community.

Through these actions, the LMS reinforces its dedication to sustainability and the responsible stewardship of resources for a greener future.

Lesley Campbell
Society Governance Officer

Mathematical Notes on Interference Protection Criteria for the Fixed-Satellite Service

IAN FLOOD, GLYN CARTER AND JOHN PARKER

Interference protection criteria are required by engineers conducting radio spectrum coexistence studies, during which the feasibility of coexistence between different radio services and technologies are investigated. Some complex, and ongoing, discussions within the International Telecommunication Union have focused on the development of protection criteria for the fixed-satellite service. In this article, we set out to explain some aspects of these discussions from a mathematical perspective.

Introduction

There have been extended discussions within the International Telecommunication Union Radiocommunication Sector's (ITU-R) Working Party 4A on the development of interference protection criteria for fixed-satellite service (FSS) receivers; the criteria, once agreed upon, would be used in radio spectrum coexistence studies conducted within the ITU's remit. These discussions have been complex, chaotic and discordant, with proposals often unsupported by a strong rationale or mathematical rigour. In this article, we discuss some of the analysis undertaken by Transfinite Systems on behalf of the mobile communications industry association GSMA.

An investigation into coexistence between services in the same frequency band is a *sharing* study, but when we focus on coexistence between services in adjacent frequency bands, it is a *compatibility* study.

In radio engineering studies where events are modelled in the time domain, long-term interference is often associated with interference protection criteria that can be exceeded for a permissible 20% of the time and short-term interference, which is a higher level of interference, with criteria that can be exceeded for $\leq 1\%$ of time; see [5] for some elaboration of this point. Hence, separate interference protection criteria are developed for long-term and short-term interference problems, and the discussions within ITU have been concerned with both. However, because of the complexities of these discussions, time-invariant criteria, which are appropriate for static analyses where no exceedance is permitted, have also been under discussion and the basis for some proposals. In addition, protection criteria associated with interference sourced to an adjacent frequency band are often based on (or taken directly from) established time-invariant criteria.

Within radio spectrum management, a radio service (such as FSS) has 'primary' status when it has priority use of a frequency band, and services are 'co-primary' when more than one service has primary status in a frequency band. In this article, we discuss the long-term interference protection criteria associated with co-primary sources of interference and the criteria associated with sources operating in adjacent frequency bands. We set out a mathematical analysis of different proposals and provide an explanation of the impact of these criteria at the FSS receiver.

When modelling coexistence problems in the radio interference environment, it is important to select appropriate interference protection criteria. These criteria can be formulated in several ways (see the discussion in [1]), including as an interference-to-noise ratio I/N expressed in decibels and its associated percentage time, which is the percentage of the time that I/N may be exceeded.

We denote the I/N ratios and their associated percentage times using the nomenclature n dB ($p\%$), where n is the I/N ratio expressed in decibels and ($p\%$) is the percentage time that I/N may be exceeded. Hence, in this paper, for criteria that may not be exceeded for any percentage of the time, we set $p = 0\%$.

The discussion on long-term interference protection criteria has been complicated by two different interpretations of the interference apportionment scheme specified in Recommendation ITU-R S.1432 [2]. Although [2] sets out apportionments for time-invariant interference incident to FSS receivers in space, discussions on the development of interference protection criteria for time-varying interference will often reference these apportionments.

Here we set out a mathematical description of these two interpretations of [2] that is consistent with the

notes set out in [6]. For co-primary sources of interference, the first interpretation, which we label the *conventional* interpretation, leads to $I/N = -10$ dB (20%), while a second interpretation, which we label the *alternative*, has led to a proposal for $I/N = -10.5$ dB (20%). This 0.5 dB difference in I/N may appear trivial, but we will show that the two I/N ratios rest on radically different assumptions regarding the acceptable levels of interference incident to an FSS receiver.

The interference margin M is the degradation of receiver noise due to interference, expressed in decibels. The conventional interpretation treats the apportionments as percentage degradations of receiver noise due to various sources of interference, leading to overall values for M of around 1 or 1.2 dB, which are consistent with those mentioned in [2]. The alternative interpretation models receiver noise such that interference is included in the noise value and treats the apportionments as percentages of the noise that are attributable to various sources of interference. Importantly, this second method leads to overall values for M of 1.37 and 1.67 dB, which are significantly higher than, and so inconsistent with, those given in [2].

The discussions have also considered the interference protection criteria used in adjacent-band compatibility studies. Here, the scheme in [2] is often cited as the basis for $I/N = -20$ dB (0%), which is regarded as unnecessarily conservative by representatives of emerging technologies and services. We discuss alternative formulations of the apportionments that allow for a significant relaxation of this criterion for a relatively small increase in the overall interference margin at the FSS receiver.

Conventional and alternative interpretations of recommendation ITU-R S.1432

The apportionment scheme for interference from all sources of interference incident to an FSS receiver, as set out in [2], sums these apportionments to 32% or 27% of the satellite receiver noise, dependent on FSS system frequency reuse:

- 25% for other FSS systems not practising frequency reuse
- 20% for other FSS systems practising frequency reuse
- 6% for other co-primary services

- 1% for all other sources

Hence, [2] effectively sets out two apportionment schemes: one where the FSS systems do not practise frequency reuse and a second where frequency reuse is practised. Further, [2] specifies overall interference margins of 1.2 and 1 dB for FSS systems not practising and practising frequency reuse, respectively.

The conventional method treats these apportionments as percentage degradations of noise, denoted here by $\Delta T/T$, due to various sources of interference.

We can calculate the interference margin M , expressed in decibels and associated with $\Delta T/T$, using

$$M = 10 \log_{10} \left(1 + \frac{\Delta T/T}{100} \right). \quad (1)$$

Then, I/N is obtained via

$$I/N = 10 \log_{10} \left(10^{M/10} - 1 \right). \quad (2)$$

Tables 1 and 2 show values for M and I/N calculated for each individual apportionment in [2] and for the sum of these apportionments. We can see that the overall interference margins of 1.21 and 1.04 dB are consistent with [2].

Table 1. M and I/N (no frequency reuse)

Source	$\Delta T/T$	M	I/N
FSS	25	0.97	-6.02
Co-primary	6	0.25	-12.22
Others	1	0.04	-20
All	32	1.21	-4.95

Table 2. M and I/N (frequency reuse)

Source	$\Delta T/T$	M	I/N
FSS	20	0.79	-6.99
Co-primary	6	0.25	-12.22
Others	1	0.04	-20
All	27	1.04	-5.69

Importantly, [2] sets out apportionments for time-invariant interference. Hence, $p = 0\%$ for all the I/N ratios derived from these apportionments. However, [2] is often cited as the basis for criteria specified for use in studies where interference is time-varying. The interference apportionments in Tables 1 and 2 lead

to an $I/N = -12.2$ dB (0%) for co-primary sources of interference, and this criterion is sometimes reformulated as $I/N = -10$ dB (20%) in ITU managed studies; see [4, 3], for example. However, during the Working Party 4A discussions the question has been raised as to whether there exists a sound basis for such a reformulation.

The alternative method models receiver noise such that interference is included and treats the apportionments as percentages of noise attributable to various sources of interference. An individual apportionment for a particular source of interference is denoted here by p_a and the sum of these apportionments by p_I .

The I/N associated with an apportionment is given by

$$I/N = 10 \log_{10} \left(\frac{p_a}{100} \right) - 10 \log_{10} \left[1 - \left(\frac{p_I}{100} \right) \right]. \quad (3)$$

Note that the I/N associated with the sum of these apportionments is obtained when p_a is substituted by p_I in the first logarithmic term.

Having calculated the I/N ratios, the degradation of noise $\Delta T/T$ for each apportionment can be calculated:

$$\Delta T/T = 100 \times 10^{(I/N)/10}. \quad (4)$$

Tables 3 and 4 set out the I/N ratios, $\Delta T/T$ and M for each individual apportionment and the total values obtained when considering the sum of apportionments.

Table 3. I/N , $\Delta T/T$ and M (no frequency reuse)

Source	p	I/N	$\Delta T/T$	M
FSS	25	-4.35	36.76	1.36
Co-primary	6	-10.54	8.82	0.37
Others	1	-18.33	1.47	0.06
All	32	-3.27	47.06	1.67

Table 4. I/N , $\Delta T/T$ and M (frequency reuse)

Source	p	I/N	$\Delta T/T$	M
FSS	20	-5.62	27.4	1.05
Co-primary	6	-10.85	8.22	0.34
Others	1	-18.63	1.37	0.06
All	27	-4.32	36.99	1.37

Clearly, the overall interference margins of 1.67 and 1.37 dB are not consistent with those given in [2].

We can see that an $I/N = -10.5$ dB criterion for co-primary sources is associated with $p_a = 6\%$ and a scheme where the FSS system does not practise frequency reuse. In the discussions within the ITU, this I/N ratio has been associated with 20% of time but with no explanation of how this criteria has been reformulated.

Adjacent-band interference

An important aspect of these discussions has been around the overall value of M available at the FSS receiver. That is, how much degradation of noise at the receiver, due to interference, is acceptable to the satellite operator. This feature of the discussion comes into sharp focus for adjacent-band compatibility.

A major concern for emerging technologies when seeking solutions to spectrum-sharing and compatibility problems is the long-term protection criterion of $I/N = -20$ dB (0%), which is often cited for use in adjacent-band compatibility studies. Here, we consider the apportionment in [2] of 1% for other sources of interference, which is the basis for this criterion and practical adjustments that could deliver a significant relaxation for a relatively small increase in the overall value of M at the victim receiver.

Following the conventional interpretation of [2], a $\Delta T/T = 1\%$ apportionment leads to $I/N = -20$ dB (0%) (as shown in Tables 1 and 2). However, if we increase this apportionment to 3.16%, say, this results in a 5 dB relaxation of the apportionment's I/N and an increase in the overall interference margin of just 0.07 dB. This adjustment is set out in Table 5.

Table 5. Example adjustment (frequency reuse)

Source	$\Delta T/T$	M	I/N
FSS	20	0.79	-6.99
Co-primary	6	0.25	-12.22
Others	3.16	0.14	-15.00
All	29.16	1.11	-5.35

For illustrative purposes, these changes can be presented in terms of the percentage increase in I/N for other services and the consequent increase in the overall interference margin M :

$$\Delta \text{dB} (\%) = 100 \times \left(10^{\Delta \text{dB}/10} - 1 \right), \quad (5)$$

where ΔdB is a change to a value expressed in decibels. Table 6 summarises this example adjustment in terms of percentage change.

Table 6. Example adjustment ΔdB (%)

Parameter	ΔdB	ΔdB (%)
I/N for other sources	5	216.23
Overall M	0.07	1.62

Our 5 dB relaxation delivers ΔdB (%) = 216.23%. However, the 0.07 dB change to the overall value of M at the FSS receiver leads to a ΔdB (%) of just 1.62%. These are important observations because we show how significant change can be introduced with very little impact on the overall interference margin M when our start point is a stringent interference protection criterion. In this case, prior to our adjustment, the criterion maintains an interference threshold for other sources at a level equal to 1/100th of the value of receiver noise.

Based on this understanding, in Table 7 we set out a second example adjustment with percentage changes shown in Table 8. Here, a 7.8 dB relaxation of the I/N associated with other services results in an overall increase in M of 0.17 dB. This adjustment delivers a 502.56% increase in the allowable interference apportioned to other services and a 3.99% increase in the overall value of M .

Table 7. Second example adjustment (frequency reuse)

Source	$\Delta T/T$	M	I/N
FSS	20	0.79	-6.99
Co-primary	6	0.25	-12.22
Others	6	0.25	-12.22
All	32	1.21	-4.95

Table 8. Second example adjustment ΔdB (%)

Parameter	ΔdB	ΔdB (%)
I/N for other sources	7.8	502.56
Overall M	0.17	3.99

With this adjustment, the criterion for other services is now exactly equal to that specified for co-primary sharers. We have assumed frequency reuse in the satellite network, but note that the overall interference margin M is now exactly equal to the higher value of M given in [2] and associated with no frequency reuse.

Discussion

The two different interpretations of [2] have greatly complicated the discussions on the specification of appropriate interference protection criteria for FSS receivers. This paper has set out a mathematical explanation of both interpretations within the context of an interference apportionment scheme in which the derived I/N ratios cannot be exceeded.

The most striking observation, in relation to this analysis, is that the alternative interpretation requires significantly larger interference margins at the FSS receiver than those given by the conventional interpretation. Yet, when emerging services and technologies press for relaxed interference protection criteria to be used in sharing and compatibility studies, it is often stated that the consequent increase in the overall interference margin at the FSS receiver cannot be tolerated.

We have set out calculations and example adjustments of the interference protection criteria to be used in adjacent-band compatibility studies. This is based on a mathematical analysis of the impact of the proposal on the overall interference margin M at the victim receiver. A very significant change can be accommodated via a relatively modest increase in M .

The interference apportionments given in [2] are associated with time-invariant interference; that is, the I/N ratios derived from these apportionments cannot be exceeded. However, it is generally the case that in coexistence studies involving time-varying interference sources, long-term interference protection criteria can be exceeded for 20% of the time [5].

It is also the case that [2] is sometimes cited as the basis for the reformulated interference protection criteria used in studies that are investigating time-varying interference. The criteria $I/N = -10$ dB (20%) is an established reformulation of $I/N = -12.2$ dB (0%) [4, 3].

The rationale for this reformulation, which appears to rest, at least in part, on information within [2], has been questioned: the recommendation includes a graph of I/N versus percentage time. The graph shows an extrapolation of satellite receiver I/N and percentage time values, which include $I/N = -10$ dB (20%) and $I/N = -12$ dB (0%).

Hence, the time-invariant interference apportionment scheme in [2] is cited and utilised, while the derived I/N ratio associated with the apportionment

for co-primary sources of interference is subject to reformulations when used in time-varying studies; these are presented with either a disputed rationale or no rationale, as per the alternative interpretation of [2], which has been widely accepted at the time of writing.

Note that when considered in isolation from the apportionment scheme in [2], the two reformulations of I/N associated with the co-primary interference apportionment result in individual interference margins M of 0.41 and 0.37 dB for the conventional and alternative interpretations of [2], respectively. Although the underlying assumptions deliver significant differences when the entire time-invariant apportionment scheme is considered, we found only a small difference of 0.04 dB in the individual M values when we looked at these two I/N calculations for our co-primary apportionment.

These mathematical notes are a contribution to the ongoing discussions within ITU and elsewhere. We have touched on only certain aspects of the problems under consideration, with the aim of presenting coherent explanations and a clear understanding of the impact of various proposals.

FURTHER READING

[1] I.D. Flood, J.R. Parker, G.D. Carter, A spectrally efficient service apportionment method for sharing and compatibility studies, *Wirel. Netw.* 27 (2021) 2383–2390.

[2] International Telecommunication Union, *Recommendation ITU-R S.1432-1, Apportionment of the Allowable Error Performance Degradations to Fixed-Satellite Service (FSS) Hypothetical Reference Digital Paths Arising from Time Invariant Interference for Systems Operating below 30 GHz*, 2006.

[3] International Telecommunication Union, *Report ITU-R M.2109, Sharing Studies between IMT-Advanced Systems and Geostationary Satellite Networks in the Fixed-Satellite Service in the 3,400–4,200 and 4,500–4,800 MHz Frequency Bands*, 2007.

[4] International Telecommunication Union, *Report ITU-R S.2368-0, Sharing Studies between International Mobile Telecommunication-Advanced Systems and Geostationary Satellite Networks in the Fixed-Satellite Service in the 3,400–4,200 MHz and 4,500–4,800 MHz Frequency Bands in the WRC Study Cycle Leading to WRC-15, S Series, Fixed Satellite Service*, 2015.

[5] International Telecommunication Union, *Recommendation ITU-R SM.1448, Determination of the*

Coordination Area around an Earth Station in the Frequency Bands between 100 MHz and 105 GHz, 2019.
[6] Transfinite Systems, *Technical Note: Recommendation ITU-R S.1432 Interference Margins and I/N ratios*, 2018.



Ian Flood

Ian Flood is a consultant with Transfinite Systems, London, where his work involves modelling spectrum engineering problems in the radio interference environment, including the development of spectrally efficient methods. He is a chartered engineer and holds a PhD in graph-theoretic studies from Cardiff University.



Glyn Carter

Glyn Carter is Future Spectrum Director at the GSMA, where his work includes participation in ITU-R and CEPT on technical studies into spectrum sharing and compatibility. He has been involved in spectrum management for over 20 years and has also worked as a consultant and for mobile operators and a radio equipment vendor. He has a BSc in mathematics from Westfield College and a PhD in cryptography from Royal Holloway, University of London.



John Parker

John Parker is a founder and director of Transfinite Systems Ltd. He works as a consultant in the technical aspects of radio regulations. His work focuses on spectrum-sharing studies and the promotion of the more efficient use of the spectrum through improving numerical methods and simulations. He has a PhD in theoretical physics from the University of Manchester.

Girth of the Cayley Graph and Cayley Hash Functions

VLADIMIR SHPILRAIN

Cayley hash functions are based on a simple idea of using a pair of semigroup elements, A and B , to hash the bits 0 and 1, respectively, and then to hash an arbitrary bit string in the natural way using multiplication of elements in the semigroup. The main advantage of Cayley hash functions compared to, say, hash functions in the SHA family, is that when an already hashed document is amended, one does not have to hash the whole amended document all over again but rather hash just the amended part and then multiply the result by the hash of the original document. In this article, we survey some of the previously proposed Cayley hash functions and single out a very simple hash function whose security has not been compromised to date.

Introduction

Hash functions are easy-to-compute compression functions that take a variable-length input and convert it to a fixed-length output. Hash functions are used as compact representations, or digital fingerprints, of data and to provide message integrity. *Cryptographic hash functions* have many information-security applications, notably in digital signatures, message authentication codes and other forms of authentication. Cryptographic hash functions should satisfy the following basic security requirements:

- (1) *Collision resistance*: It should be computationally infeasible to find two different inputs that hash to the same output.
- (2) *Preimage resistance* (sometimes called *non-invertibility*): It should be computationally infeasible to find an input that hashes to a specified output.
- (3) *Second preimage resistance*: It should be computationally infeasible to find a second input that hashes to the same output as a specified input.

A challenging problem is to determine the mathematical properties of a hash function that would ensure (or at least, make it likely) that the requirements above are met.

A direction that has been gaining momentum lately is using a pair of elements, A and B , of a semigroup S to hash the bits 0 and 1, respectively. A bit string is then hashed to a product of elements in the natural way. For example, the bit string 1001011 will be hashed to the element $BAABABB$.

Since hashing a random bit string this way represents a random walk on the Cayley graph of the subsemigroup of S generated by the elements A and B , hash functions of this kind are often called *Cayley hash functions*. Note that the absence of short collisions for a Cayley hash function is equivalent to the corresponding Cayley graph having a large *girth*. The latter is defined as the length of the shortest simple circuit.

Cayley hash functions have a homomorphic property $H(XY) = H(X)H(Y)$ and the associativity property $H(XYZ) = H(XY)H(Z) = H(X)H(YZ)$ for any bit strings X , Y and Z . (Here XY means concatenation of the bit strings X and Y .) This property is useful because it allows for parallel computations when hashing a long bit string. However, a more important feature is that when an already hashed document is amended, one does not have to hash the whole amended document all over again but rather hash just the amended part and then multiply the result by the hash of the original document.

Another useful property of a Cayley hash function is that, unlike some other hash functions, you do not have to know the length of the bit string to be hashed upfront; you can hash ‘as you go’.

While the high-level idea of Cayley hashing is definitely appealing, the choice of the platform semigroup S and the two elements $A, B \in S$ is crucial for security and efficiency. There have been many proposals based on matrix semigroups in $GL_2(\mathbb{F})$ for various fields \mathbb{F} , in particular for $\mathbb{F} = \mathbb{F}_p$. This is because Cayley graphs of 2-generator semigroups in $GL_2(\mathbb{F}_p)$ often have a large girth, as has been shown by several authors. See, e.g., [1, 2, 3, 4].

Cayley graphs of (semi)groups in $GL_n(\mathbb{F}_p)$ with $n > 2$ have been considered, too (see, e.g., [6]), but we will focus here on $n = 2$, one of the reasons being a smaller hash size. For example, if p is a 256-bit prime, then any matrix from $GL_2(\mathbb{F}_p)$ has a total size of up to 1,024 bits, which is common for standard hash functions these days.

Specific platform (semi)groups

The first proposal for a Cayley hash function was due to Zémor [12]. The matrices used, considered over \mathbb{F}_p , were

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}.$$

This proposal was successfully attacked in [10]. Specifically, it was shown that this hash function is not preimage resistant.



Figure 1. Gilles Zémor, Université de Bordeaux

The most cited proposal is what has become known as the Tillich-Zémor hash function [11]. Their matrices were

$$A = \begin{pmatrix} \alpha & 1 \\ 1 & 0 \end{pmatrix},$$

$$B = \begin{pmatrix} \alpha & \alpha + 1 \\ 1 & 1 \end{pmatrix}.$$

These matrices are considered over a field defined as $R = \mathbb{F}_2[x]/(\mathfrak{p}(x))$, where $\mathbb{F}_2[x]$ is the ring of polynomials over \mathbb{F}_2 , $(\mathfrak{p}(x))$ is the ideal of $\mathbb{F}_2[x]$ generated by an irreducible polynomial $\mathfrak{p}(x)$ of degree n (typically, n is a prime, $127 \leq n \leq 170$) and α is a root of $\mathfrak{p}(x)$.

The reason for selecting such a ‘fancy’ field was probably to specifically avoid the attack in [10].



Figure 2. Jean-Pierre Tillich, Centre Inria de Paris

Similar later proposals and attacks (some of them targeted at finding collisions; some targeted at finding a preimage) have been suggested over the years. See, e.g., [9] for a list of relevant references.

A simple yet fruitful idea for avoiding short collisions is to use a pair of 2×2 matrices, A and B ,

over \mathbb{Z} that generate a free semigroup in $GL_2(\mathbb{Z})$ and then reduce the entries modulo a large prime p to get matrices over \mathbb{F}_p . Since there cannot be an equality of two different products of copies of A and B unless at least one of the entries in at least one of the products is $\geq p$, this gives a lower bound on the minimum length of bit strings where a collision may occur.

Girth of the Cayley graph

The problem of bounding the girth of the Cayley graph of a 2-generator (semi)group is directly related to the security properties (specifically, to the collision resistance) of the relevant Cayley hash functions.

For matrix semigroups, if A and B generate a free sub(semi)group of $SL_2(\mathbb{Z})$, then there cannot be any relations of the form $u(A, B) = v(A, B)$ in $SL_2(\mathbb{Z}_p)$ unless at least one of the entries of the matrix $u(A, B)$ or $v(A, B)$ is at least p . Thus, if the largest entry in a product of n matrices is of the size $O(s^n)$, then the girth of the Cayley graph of the sub(semi)group of $SL_2(\mathbb{Z}_p)$ generated by A and B is $O(\log_s p)$. This (maximal) growth rate s is called the *joint spectral radius* of the pair (A, B) of matrices and has been studied (in greater generality) a lot. See, e.g., [5].

We are interested in having the joint spectral radius of (A, B) as small as possible to have a larger girth of the corresponding Cayley graph. To that end, let us consider pairs of matrices $(A(k), B(m))$, where

$$A(k) = \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}, \quad B(m) = \begin{pmatrix} 1 & 0 \\ m & 1 \end{pmatrix}.$$

The Cayley graph of the (semi)group generated by these two matrices (considered over \mathbb{Z}_p), especially when $m = k$, has been extensively studied in the literature. See, e.g., [1, 4] and references therein.

When considered over \mathbb{Z} , the joint spectral radius of the pair of matrices $(A(k), B(k))$ for $k \geq 1$ has been computed, in particular, in [2] and [3]. In fact, [2] gives explicit formulae for the largest entry in a product of n copies of $A(k)$ and $B(k)$. As expected, the smallest joint spectral radius is achieved when $k = 1$ (and is equal to $\frac{1}{2}(1 + \sqrt{5}) \approx 1.618$), but as we mentioned before, the corresponding Cayley hash function was successfully attacked in [10].

Therefore, [2] proposed using the Cayley hash function corresponding to the pair of matrices

$(A(2), B(2))$, where the joint spectral radius is $1 + \sqrt{2} \approx 2.414$. It is worth mentioning that powers of the matrix $A(2)B(2)$ provide the largest entries (by absolute value) among all semigroup words in $A(2)$ and $B(2)$ of a given length.

This implies, in particular, that if p is, say, of the order of 2^{256} , then there are no collisions of the form $u(A(2), B(2)) = v(A(2), B(2))$ if both the words u and v are of length less than $201 \approx 256 \log_{2.414} 2$. This makes a ‘brute force’ search for collisions computationally infeasible.

Note that, to date, there have been no successful attacks reported against the Cayley hash function based on the matrices $A(2)$ and $B(2)$. It is also worth mentioning that this hash function has successfully passed all the pseudorandomness tests in the NIST Statistical Test Suite [8].

Cayley hashing with cookies

In [9], the authors introduced an enhancement of Cayley hashing that they called ‘Cayley hashing with cookies’, the terminology being borrowed from the theory of random walks in a random environment. The authors argue that this enhancement does not affect the collision-resistance property. Moreover, it makes the hash function more preimage resistant. The homomorphic property is ‘almost preserved’. That is, it is preserved upon minor padding.

A ‘cookie’ is a place in the Cayley graph where some of the parameters of a random walk change in a specific way. There is a lot of flexibility in positioning cookies in the Cayley graph as well as in choosing a particular way a cookie affects parameters of a random walk on the Cayley graph. Below is an example (from [9]) of an instantiation of this general idea.

Let A , B and C be 2×2 matrices. Let u be a bit string of arbitrary length. Then, to hash u , going left to right:

- (1) If the current bit is 0, then it is hashed to the matrix A . If the current bit is 1, then it is hashed to the matrix B .
- (2) If there are three 1 bits in a row (a ‘cookie’), then all following 1 bits will be hashed to the matrix C until there are three 0 bits in a row, in which case hashing the 1 bit is switched back to the matrix B . For example, the bit string

1100111 01011 00011 will be hashed to the matrix $BBAABBBACACCAAABB$.

In [9], the recommended particular matrices were

$$A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, C = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}.$$

These matrices generate a free semigroup when considered over \mathbb{Z} . It was shown in [9] that the joint spectral radius of the triple of matrices of (A, B, C) as above is

$$\frac{7}{2} + \frac{3\sqrt{5}}{2} \approx 2.618.$$

Directions for further research

Most of the theoretical results (if not all of them) on the joint spectral radius of matrices are relevant to matrices with non-negative entries. See, e.g., [5]. However, having in mind our goal of minimising the joint spectral radius, there is nothing wrong with using matrices some of whose entries are negative.

An obvious candidate here would be the pair $(A(2), B(-2))$, where

$$A(2) = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}, B(-2) = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}.$$

We do not know what the joint spectral radius of this pair of matrices is, but computer experiments suggest that it is $\sqrt{2 + \sqrt{3}} \approx 1.93$, so it is considerably smaller than $1 + \sqrt{2} \approx 2.414$, the joint spectral radius of the pair $(A(2), B(2))$. It would be good, though, to establish this result theoretically. Also note that, again based on computer experiments, it appears that powers of the matrix $A(2)^2 B(-2)^2$ provide the largest entries (by absolute value) among all semigroup words in $A(2)$ and $B(-2)$ of a given length.

Another research direction related to Cayley hashing (with matrices A and B) is motivated by the fact that since bit strings that are hashed in real-life applications can be considered random, one might look at the length of ‘generic’ simple circuits of the relevant Cayley graph instead of looking for the length of the shortest simple circuit. To that end, one can consider products of n matrices, where each factor is either A or B with probability $1/2$ and see how the largest entry in such a product grows when n goes to infinity. This yields interesting connections to the theory of stochastic processes. See, e.g., [7].

FURTHER READING

- [1] J. Bourgain, A. Gamburd, Uniform expansion bounds for Cayley graphs of $SL_2(\mathbb{F}_p)$, *Ann. Math.* 167(2) (2008) 625–642.
- [2] L. Bromberg, V. Shpilrain, A. Vdovina, Navigating in the Cayley graph of $SL_2(\mathbb{F}_p)$ and applications to hashing, *Semigr. Forum* 94 (2017) 314–324.
- [3] S. Han, A.M. Masuda, S. Singh, J. Thiel, Maximal entries of elements in certain matrix monoids, *Integers* 20 (2020), A31.
- [4] H.A. Helfgott, Growth and generation in $SL_2(\mathbb{Z}/p\mathbb{Z})$, *Ann. Math.* 167(2) (2008) 601–623.
- [5] R. Jungers, *The Joint Spectral Radius: Theory and Applications*, in *Springer Lecture Notes in Control and Information Sciences*, vol. 385 (2009).
- [6] C. Le Coz, C. Battarbee, R. Flores, T. Koberda, D. Kahrobaei, Post-quantum hash functions using $SL_n(\mathbb{F}_p)$. Preprint, 2024, arxiv.org/abs/2207.03987.
- [7] M. Pollicott, Maximal Lyapunov exponents for random matrix products, *Invent. Math.* 181 (2010) 209–226.
- [8] A. Rukhin et al. *A Statistical Test Suite for Random and Pseudorandom Num-*

ber Generators for Cryptographic Applications. NIST SP 800-22 Rev. 1, 2010, csrc.nist.gov/pubs/sp/800/22/r1/upd1/final.

[9] V. Shpilrain, B. Sosnovski, Cayley hashing with cookies. Preprint, 2024, arxiv.org/abs/2402.04943.

[10] J.-P. Tillich, G. Zémor, Group-theoretic hash functions, in *Lecture notes in Computer Science*, vol. 781, Springer, 1994, pp. 90–110.

[11] J.-P. Tillich, G. Zémor, Hashing with SL_2 , in *Lecture Notes in Computer Science*, vol. 839, Springer, 1994, pp. 40–49.

[12] G. Zémor, Hash functions and graphs with large girths, in *Lecture Notes in Computer Science*, vol. 547, Springer, 2001, pp. 508–511.



Vladimir Shpilrain

Vladimir is a professor of mathematics at the City College of New York and a member of the doctoral faculty at the CUNY Graduate Center.

His main research interests are in information security, complexity of algorithms, and combinatorial and computational group theory.

AI Tools: The Facts of Licensing

SIMON BUCKMASTER

“We’re all working on them. Whether people are saying they are or not. We’re all working on them.”

This was the moment that a panellist, the commercial director of a major university press, first said aloud what everyone else in the publishing conference room already knew. That most university presses and publishers are working on deals to license published academic research for use with AI tools. In this article, I aim to cover everything you need to know about this latest development in the publishing world.

In the spring of 2024, news sources reported that Wiley — the Society’s main journal publishing partner — was one of two publishers that had signed deals to license published academic content for training generative AI (GenAI) tools [1].

News of these deals was met with some negative comment, but university presses and publishers have quietly pressed on with negotiating such licences [6].

Researchers in applied subjects (engineering, biomedicine, etc.) are accustomed to their research being sold to corporate customers. They may even operate commercial spin-offs themselves. But this is the first time that many pure mathematicians have faced the idea that their research publications could be exploited by commercial companies.

In this article, I will explain the motivations for publishers to strike these deals, and whether this aligns with researchers’ interests. I will explain the basics of licensing and talk about how published research might be used with different types of GenAI tools. I will cover whether this affects open access content or work on arXiv.org. And I will answer the important question of whether authors can opt out.

What has happened so far?

To develop GenAI tools, technology companies need to process content at enormous scale. In their rush to compete with each other, these technology firms are alleged to have harvested and used any and all available material, ignoring the copyright of creators or the licensing conditions of publishers [7].

The material used includes books, newspaper articles, music, images and video. In the academic space, this means books, journal articles and even material on preprint servers like arXiv.org.

To defend their intellectual property, rights holders have launched at least 40 legal cases alleging copyright infringement by technology companies [5]. These cases are now making their way through the courts.

To strengthen their legal cases and undermine a ‘fair use’ defence, rights holders have concluded that they need to provide legal and robust frameworks for technology companies to access content, with appropriate terms and conditions applied.

Essentially, if rights holders don’t provide a legal way for technology companies to access and use their content through licensing, it is believed to be more likely that judges will rule that the companies are free to harvest published content and use it as they see fit.

Rights holders that do negotiate licences will be able to generate an income and apply conditions on the usage that align with their authors’ interests [2]. One important condition is that when a GenAI tool reproduces more than minimal licensed content, there are appropriate attributions to the sources used.

What is licensing?

Licences govern the access and usage rights for published works. In the context of journals, you probably know licences as subscriptions. But if making work available on an open access basis, you may also have encountered Creative Commons licences.

Academic institutions buy licences from publishers and university presses so that their faculty can access and use published articles. Publishers also sell licences to commercial companies, which use the research to further their own R&D activities and develop their products.

Commercial companies are most interested in academic content from applied subjects, those subjects where translating research into commercially viable products is more straightforward. However the publishing marketplace has consolidated into big package deals, and corporate licences now tend to include all content from a university press or publisher rather than individual titles.

This means that, even if it isn't of interest to most companies, the mathematical research published in our journals has been licensed to commercial customers for many years prior to the current debate around GenAI.

In the last decade, publishers' licences have already developed to cover uses of research content beyond just the human reader.

Text- and data-mining are where an automated tool processes structured text and data to detect patterns or relationships and to facilitate better searching and discoverability. University presses and publishers have permitted text- and data-mining for non-commercial purposes to further academic research, while also selling licences to companies to do the same for commercial purposes.

Licensing funds the Society

Last year, our journals and books contributed more than 40% of the Society's total income for the year, primarily from licensing subscription journal articles to academic institutions and commercial companies.

Our contracts with our publishing partners mean that the Society will generate additional income for any AI licensing deal, which includes research published by the Society.

We use all surplus income from our publications to support mathematicians and mathematics research. This includes funding mathematics conferences and awarding research grants and prizes.

What do we mean by AI licensing?

AI licensing covers more than just a major tech firm training the latest version of a large language model (LLM). This is only one of three broad use cases [3]:

- (1) Foundational training of an LLM
- (2) Fine-tuning an LLM
- (3) Retrieval-augmented generation

For each of these uses, there are different types of customer, and different sorts of academic content are preferred.

1. Foundational training of an LLM

Foundational training is what most people will think of when they read about AI licensing. It is the creation of the set of weights at the heart of an LLM.

Weights are variables that determine the strength of the relationship between different words or fragments of text (called tokens). The weights in an LLM are used to predict what word to return next when generating a response to a prompt. To determine the weights that a model will use, the developers run trillions of sources of text through neural networks. Meta's Llama 3.1 model had 405 billion weights in it.

Once training is complete, the original sources of text are not retained verbatim within the LLM. All that remains is the set of weights, which are the result of processing a huge number of original sources.

The huge costs involved in foundational LLM training mean that only a small number of big technology firms (Open AI, Microsoft, Google, Meta, etc.) are able to do this work.

Publishers are finding that these customers are primarily interested in book content and have very little interest in journal content.

Academic books are comprehensive, well edited and accessible while still being at a high-Lexile level, which is perfect for foundational LLM training. Journal articles are thought to be too specific and technical to be useful.

2. Fine-tuning an LLM

Fine-tuning is when a general LLM is optimised for a specific purpose. The developers give a general-purpose LLM a large, but curated, set of high-quality sources focused on a specific subject or domain. The fine-tuning process adjusts the weights in the original LLM so that it will produce better results for the specific purpose or subject area.

As with foundational training, all that remains in the LLM after fine-tuning is the new set of weights. The original source material is not retained within the model.

As the starting point is an existing LLM, fine-tuning is not restricted to the biggest technology companies. Smaller companies and research groups are able to license a commercial LLM, or use an open source LLM, and then optimise it for their purposes. Despite the specialisation, fine-tuning still requires a large amount of source material.

For customers interested in licensing academic content for fine-tuning, books are still the main focus. There is some growing interest in journal articles for fine-tuning too, as these include more specific and technical use of language than books.

3. Retrieval-augmented generation

Retrieval-augmented generation (RAG) is when an LLM (likely after it has been optimised for a specific domain) is given access to a specific set of content as a reference. The AI tool will use the language capabilities of the LLM and combine it with the reference content to generate useful outputs.

You may already have seen RAG in action because Google search results are now often preceded by an answer generated by Google's Gemini tool, which uses all the content that Google indexes as a reference.

Like fine-tuning, RAG can be performed by smaller companies as well as the big technology firms. RAG is expected to become an important tool for academic research groups and companies in research-intensive industries.

The most interest in licensing academic journal articles comes from research-focused applications of RAG. In these cases, the companies are looking for highly technical sources of information that are frequently updated with new knowledge rather than static books.

As the results of this type of activity are likely to make significant use of the reference content, it is important that the generated responses include attribution and links to the original sources.

Can I opt out?

Firstly, there is an important caveat. In a world where it is reasonable to assume that any published work

has been used to train GenAI tools, the question of opting out gains practical relevance only if technology firms can be made to respect intellectual property rights.

When asked about opt-outs, publishers point out that authors are not asked to consent individually to any other types of licences nor are they given the opportunity to pick and choose which companies or institutions may be sold access to their work.

There is also a difficulty of how to determine the types of AI licensing that authors might wish to opt out of. Some researchers may wish to opt out of foundational and fine-tuning of LLMs but be completely comfortable with RAG.

Publishers also question the practicality of managing individual opt-outs at scale. To date, Cambridge University Press & Assessment is the only major university press or publisher that has announced that it will seek the consent of 20,000 book authors for AI licensing [9].

What does this mean for the Society's publications?

Customers wanting to license academic content for this purpose are primarily interested in books. Our two book series, *LMS Lecture Notes* and *LMS Student Texts*, are published by Cambridge University Press & Assessment, which has announced that it will seek author consent for AI licensing.

Customers wishing to pair the language power of a trained LLM with a trusted set of reference materials will want to license both journals and books. Our journals are published by Wiley, which is actively licensing content for use with AI, although its customers are most interested in applied subjects.

What about open access and preprints?

Everything discussed so far applies to academic books for sale and to research content published in journals on a subscription basis.

Over the last 20 years, advocates of the open access movement have pushed for research to be freely available and reusable. There has been remarkable progress to this end, and, last year, more than 50% of the research published in our journals was published as gold open access with a Creative Commons licence.

Research funders have strongly encouraged, or even mandated, the use of the Creative Commons Attribution (CC BY 4.0) licence for open access publications. Thus, the CC BY licence has become the most commonly used licence for open access. The CC BY licence enables reusers to distribute, remix, adapt and build upon the material in any medium or format, so long as attribution is given to the creator [4].

This licence makes no distinction about the type of person or organisation seeking to reuse the work, or how they intend to use it. A work published under a CC BY licence is protected only by the requirement of attribution.

Whether the attribution protection will prevent CC BY licensed works from being used for foundational training or fine-tuning of an LLM has not yet been litigated. But, as each work is only one of trillions of sources that produce the model weights, attribution is impractical in the extreme.

If RAG tools cite any reference materials that contribute significantly to their generated answers, then use of CC BY works would be completely within the terms of the licence.

Only Creative Commons licences that include the noncommercial (NC) or no derivatives (ND) clauses would seem to offer any protection against use with AI tools. Again, this has yet to be litigated.

Creative Commons are now exploring adding ‘preference signals’, so that creators can express their preference about whether their work is used to train GenAI tools [8]. However, these signals are not intended to be legally enforceable.

The default arXiv licence says nothing about the reuse of material, which the arXiv team consider to be sufficient protection. The arXiv continues to allow bulk downloading of content from its servers.

UK Copyright and AI Consultation

The UK government is consulting on creating an exception to copyright law for AI technology firms. If implemented, this exception would put the onus on creatives and rights holders to actively opt out of the unrestricted use of their work by AI technology firms.

LMS members may wish to support the Creative Rights in AI Coalition (www.creativerightsinai.co.uk) and consider writing to their MP to oppose this copyright exemption.

FURTHER READING

- [1] M. Battersby (2024) Wiley set to earn \$44m from AI rights deals, confirms ‘no opt-out’ for authors, thebookseller.com [Accessed 01/12/2024].
- [2] M. Battersby (2024) Sage confirms it is in talks to license content to AI firms, thebookseller.com [Accessed 01/12/2024].
- [3] T.A. Carpenter (2024) Wiley leans into AI. The Community should lean with them, scholarlykitchen.sspnet.org [Accessed 01/12/2024].
- [4] Creative Commons, About CC licenses, creativecommons.org [Accessed 01/12/2024].
- [5] George Washington University, DAILE — the database of AI litigation, blogs.gwu.edu [Accessed 01/12/2024].
- [6] Ithaka S+R, Generative AI Licensing Agreement Tracker, sr.ithaka.org [Accessed 01/12/2024].
- [7] C. Metz, C. Kang, S. Frenkel, S.A. Thompson, N. Grant (2024) How tech giants cut corners to harvest data for A.I. nytimes.com [Accessed 01/12/2024].
- [8] R. Ross, Six insights on preference signals for AI training, creativecommons.org [Accessed 01/12/2024].
- [9] M. Spanoudi (2024) Cambridge University Press & Assessment writes to 20k authors for AI licensing ‘opt-in’, thebookseller.com [Accessed 01/12/2024].



Simon Buckmaster

Simon Buckmaster is the Society’s Head of Academic Publications. With a background in astrophysics, Simon has spent the last 14 years publishing journals in applied physics, materials science and biomedicine. Simon joined the LMS staff team in the summer of 2023 and leads the Society’s Publications Team. Simon wrote this article without the use of GenAI.

Notes of a Numerical Analyst

Designer Non-uniqueness

NICK TREFETHEN FRS

I've been teaching ODEs, and we show students that some problems have non-unique solutions. For example,

$$y' = y^{1/2}, \quad y(0) = 0 \quad (1)$$

is satisfied for any $t_0 \geq 0$ by

$$y(t) = \begin{cases} 0, & t \leq t_0, \\ \frac{1}{4}(t - t_0)^2, & t \geq t_0. \end{cases} \quad (2)$$

In reversed time, this gives a consequence of Torricelli's Law of 1643: if a leaky bucket is empty, you can't tell when the last drop drained away. The reason is that the fundamental existence and uniqueness theorem for $y' = f(t, y)$ assumes that f is Lipschitz continuous with respect to y , which does not hold for $f(t, y) = y^{1/2}$.

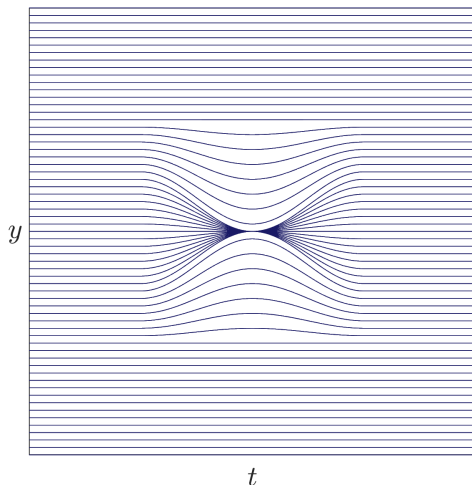


Figure 1. A flow field with a pinch point gives non-unique solutions to an ODE at that point.

What may seem surprising for an ODE becomes elementary when you plot the field of solution curves $(t, y(t))$. A point of non-uniqueness is just a point where the curves pinch together, as in Fig. 1. The set of all these trajectories defines an ODE in which $f(t, y)$ is simply the value $y'(t)$ at each point.

Once we note that non-uniqueness is a matter of pinch points, we can design ODEs with non-

uniqueness wherever we like. One idea, going back to Lavrentieff and Hartman [1, 2], is to have a dense infinity of pinch points with decreasing spatial scales. Figure 2 suggests the first step of such a construction.

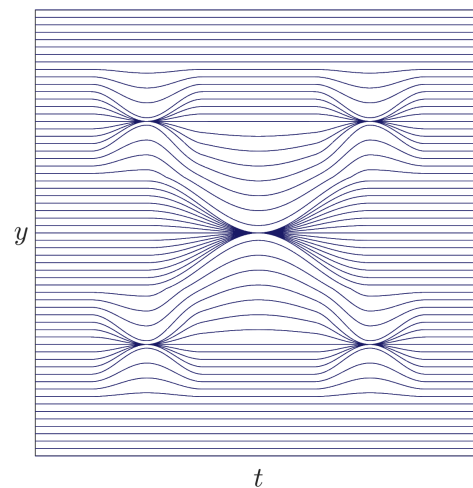


Figure 2. More points of non-uniqueness can be added.

Taking the process to the limit, we see that there exists an ODE $y' = f(t, y)$ with the property that for every initial point (t_0, y_0) , there is more than one solution on every interval $[t_0, t_0 + \varepsilon]$.

FURTHER READING

- [1] P. Hartman, A differential equation with non-unique solutions, *Am. Math. Mon.*, 70 (1963) 255–259.
- [2] M. Lavrentieff, Sur une équation différentielle du premier ordre, *Math. Z.*, 23 (1925) 197–209.
- [3] L.N. Trefethen, Á. Birkisson, T.A. Driscoll, *Exploring ODEs*, SIAM, 2017.



Nick Trefethen

Trefethen is Professor of Applied Mathematics in Residence at Harvard University.

Mathematics News Flash

Jonathan Fraser reports on some recent breakthroughs in mathematics.

Knots inside fractals

AUTHORS: Joshua Broden, Malors Espinosa, Noah Nazareth, Niko Voth

ACCESS: arxiv.org/abs/2409.03639

Fractals are sets that exhibit complexity on arbitrarily small scales. A particularly well-known fractal is the *Menger sponge*, a self-similar fractal set living in 3-dimensional space that is built by iterating a simple procedure based on cubes. More precisely, begin with the unit cube and divide it up into a 3×3 mesh of 27 smaller cubes of equal size. Of these 27 cubes, delete the one in the middle as well as the 6 others that share a face with the central one (but be sure to keep the boundaries of the remaining 20 cubes). Now repeat that process within each remaining cube. This creates a nested sequence of compact sets made up of lots of small cubes, and the intersection of this nested sequence is the Menger sponge. This sponge has many interesting properties and crops up as a ‘ubiquitous object’ in several areas, including quasi-conformal geometry, topology and more. It is not difficult to prove that the sponge has zero 3-dimensional Lebesgue measure but nevertheless has Hausdorff dimension $\log_3 20 = 2.7268\dots$

When Menger first introduced his eponymous sponge in 1926, he made several observations. One of these is that the sponge has a certain ‘universality property’ when it comes to curves in 3-space: all curves can be embedded homeomorphically into the sponge. (Here a ‘curve’ is anything with Lebesgue covering dimension one). This fascinating paper, appearing on the arXiv in September 2024, proves that every *knot* can be embedded in the Menger sponge. Recall that a knot is a homeomorphic embedding of the circle S^1 in \mathbb{R}^3 and that two knots are equivalent if there is an ambient isotopy between them (that is, one can be transformed continuously into the other through \mathbb{R}^3 without part of the knot passing through another part). Indeed, Menger’s result says that S^1 appears in the sponge, but it does not distinguish between distinct knots. This project grew out of a summer research workshop for high school students run by Malors Espinosa (Toronto) and yes, you guessed it, the other authors were in high school when they worked on this problem!

HADES: Fast singularity detection with local measure comparison

AUTHORS: Uzu Lim, Harald Oberhauser, Vidit Nanda

ACCESS: arxiv.org/abs/2311.04171

Suppose you want to analyse a large data set that takes the form of a (very large but finite) set of points in some high-dimensional Euclidean space; this is clearly a very hard problem with lots of computational challenges. However, suppose your data actually live on a lower-dimensional surface or manifold. Knowing this would be useful when analysing your data (dimension reduction). But what if your data come from a manifold with singularities (e.g., corners, boundaries or points of self-intersection), that is, points where the set fails to look like a piece of a lower-dimensional space. How do you detect these singularities? What does it mean to detect a singularity, given that all you have to work with is a finite set? This paper, appearing on arXiv at the end of 2023, introduces a novel, and very effective, algorithm, called HADES, which detects singularities in data sets. Very roughly speaking, HADES looks at the data set near a point, applies dimension reduction to decide what ambient dimension the data are from, deforms it into that dimension and then compares the result with a ball of the appropriate dimension using the Wasserstein metric. If the two objects are far apart, then there is a singularity. Sounds easy, right? Well, 66 pages of difficult proofs from differential geometry, optimal transport and statistics might convince you otherwise! I had the pleasure of hearing Uzu Lim present this work at the pure maths colloquium in St Andrews in November 2024.



Jonathan Fraser is a pure mathematician based in St Andrews. He works in analysis, fractal geometry and dynamics and is pictured here in Cellardyke (Fife) contemplating a swim.

Obituaries of Members

Aldric Loughman Brown: 1934 – 2024

Dr Aldric L. Brown, who was elected a member of the London Mathematical Society on 13 June 1957, died on 10 October, aged 90.



Aldric L. Brown
Credit: Monica-Shanta

Monica Shanta Brown writes: Aldric Loughman Brown, born in Bristol on 20 February 1934, was predestined to become a mathematician. He was a student at Dora Russell's progressive Beacon Hill School in Sussex during the war. As a tiny child, he chose to do mathematics every day. Aldric was the third generation in his family to study

pure mathematics at St John's College, Cambridge, achieving a double first. He then became a research student of Dr Frank Smithies, making him, according to Mathgenealogy.org, a mathematical nephew to Srinivasa Ramanujan.

With Dr Frank Smithies, Aldric embarked on the process of becoming a functional analyst, but by the time he completed his PhD thesis in 1962, *Some Problems in Linear Analysis*, he was an approximation theorist.

He was elected as a member of the London Mathematical Society in 1957. His first academic post was as lecturer in maths at the University of Nottingham. From Nottingham, Aldric moved to a post at the University of Newcastle upon Tyne, where he remained for more than 30 years.

During his time at Newcastle University, he co-wrote *The Elements of Functional Analysis* with Andrew Page. Published in 1970 and included in the *New University Mathematics* series, it is still available to buy online.

Aldric took two sabbaticals at the Ramanujan Institute in South India, first in 1969 for a year and then another in 1976 for six months. There he acquired a great love for India, especially its culture, warm

climate and the mathematical complexities of South Indian classical music.

Aldric also spent time collaborating on research at Penn State University in the US and the University of Newcastle in New South Wales, Australia. Aldric's own research students, two women and three men, were from Trinidad, India, Palestine, England and Iran.

Following his retirement from Newcastle University in his early 60s, Aldric returned to India, spending 1996 to 2001 in a research role associated with Panjab University.

Throughout his life, Aldric was a man of deep social commitment and political activism. In the early 1980s, he served for several years as the elected AUT President at Newcastle University. He was always incredibly moved by the plight of the Palestinians, and in 2007, he spent several months as a volunteer lecturer at Birzeit University in the occupied West Bank, Palestine.

Aldric was an editor of the *Journal of Approximation Theory* for about 35 years.

Following his return to the UK in 2001, he became an Honorary Research Fellow at UCL, last publishing with them in 2012 at the age of 78.

A large part of his research has concerned a variety of problems of best approximation, with his work recently described by his colleague Frank Richard Deutch of Penn State University as 'deep and difficult and impressive'.

Aldric Loughman Brown died on 10 October 2024 in South Devon, close to his daughter and granddaughter.

Death Notices

We regret to announce the following deaths:

- Professor Emeritus Ronald Brown, formerly of Bangor University, who was elected an LMS member on 14 March 1957, died on 5 December, aged 89.
- Professor Emeritus David Albert Edwards, formerly at the University of Oxford, died on 11 July, aged 95.
- Dr Roger A. Fenn, formerly of the University of Sussex, who was elected an LMS member on 16 February 1967, died on 1 January, aged 82.

UK–Japan Workshop on Nonlinear PDEs: Singularities and Asymptotic Patterns in Fluids, Chemotaxis and Geometric PDEs

30 June – 4 July 2025, ICMS Bayes Centre, Edinburgh

Website: tinyurl.com/5b2m3n3a

This week-long UK–Japan Workshop on nonlinear partial differential equations (PDEs) aims to foster bonds and research collaborations between mathematicians in the UK and Japan who are working on nonlinear PDEs. The workshop will focus on singularities and asymptotic pattern formation within three distinct but closely interconnected themes: fluids, chemotaxis and geometric PDEs. These areas have seen a surge in activity over the past decade, and the UK and Japan are leading the recent progress.

The scientific organisers of the workshop are as follows: Manuel Del Pino (University of Bath), Yoshitsugu Kabeya (Osaka Metropolitan University), Vitaly Moroz (Swansea University) and Hirokazu Ninomiya (Meiji University).

This event is partially supported by an LMS Scheme 1 grant.

Young Functional Analysts' Workshop

Location: University of Glasgow
Date: 2–4 April 2025
Website: sites.google.com/view/yfaw2025

This is a three-day annual event targeted at UK-based early career researchers in functional analysis and its adjacent fields, primarily PhD students.

The workshop will consist of five plenary talks by invited faculty and contributed talks from participants. To apply to speak at the workshop, please submit an abstract by 7 March 2025. Limited funding is available for participants. For more information and registration details, please see our website.

Postgraduate Combinatorics Conference 2025

Location: University of Glasgow
Date: 30 April – 2 May 2025
Website: sites.google.com/view/pcc2025

The PCC is a well-established event promoted by the British Combinatorial Committee. The conference is organised by and for current research students in all areas of combinatorial and discrete mathematics. The main goal of the conference is to provide an opportunity for research students to discuss their research in a relaxed environment, to gain practice at presenting their research outside their own department and to meet other researchers in their area. Registration has not yet opened, but you can register your interest on our website!

LMS Celebrating New Appointments: Representation Theory in Birmingham

Location: University of Birmingham
 Date: 19 May 2025
 Website: tinyurl.com/mpn2vekz

This one-day workshop aims to connect researchers across the UK in representation theory and related areas and is the LMS Celebrating New Appointments event for Stacey Law. The invited speakers are Karin Erdmann, Eoghan McDowell and Mark Wildon.

Registration is free for all participants. Early career researchers are particularly encouraged to attend. The workshop has some travel funding support for early career researchers and funded child-care services.

LMS–Bath Symposium on Inverse Problems and Artificial Intelligence in Medicine

Location: University of Bath
 Date: 23 June – 4 July 2025
 Website: tinyurl.com/3fyub99k

In this two-week programme, the first week is a summer school and the second week is a workshop. We will have courses on Medical Imaging (Jean Feydy), Bayesian Statistics and Uncertainty Quantification (Marcelo Pereyra), Optimal Transport (Bernhard Schmitzer), Graph-Based Machine Learning (Matt Thorpe), and Deep Learning and Applications (Michael Unser). The workshop will focus on the latest applications, methods, theory and best-practice on mathematical approaches in medicine. The week after the symposium there is a Maths4DL event on *Inverse Problems and Deep Learning* (maths4dl.ac.uk/newsevents/math4dl-conference-on-inverse-problems-and-deep-learning). More information, including how to register, can be found on the web page. The event is supported by the ICMS.

British Mathematical Colloquium and British Applied Mathematics Colloquium 2025.

Location: University of Exeter
 Date: 23–26 June 2025
 Website: sites.exeter.ac.uk/bmc-bamc2025

This major conference across pure and applied mathematics brings together students, academics and industry professionals to discuss work and exchange ideas. Topics for the workshops and mini-symposia include algebra, dynamical systems, geometry, fluid dynamics, mathematical biology, number theory, weather and climate.

The plenary speakers will be Jon Chapman (University of Oxford), Gianne Derks (Leiden University), Xue-Mei Li (EPFL/ICL), Hee Oh (Yale University), Gwyneth Stallard (Open University) and Sarah Zerbes (ETH Zurich).

This meeting is supported by the LMS, CMI, HIMR and IMA.

Society Meetings and Events

This calendar lists forthcoming Society meetings. A fuller list is given on the Society's website (lms.ac.uk/events/calendar).

May

- 14 LMS South West and South Wales Regional Meeting 2025, Cardiff
- 19 LMS Celebrating New Appointments: Representation Theory in Birmingham

June

- 4 LMS/Gresham Lecture 2025 – with Robin Wilson, London
- 25 LMS Society Meeting at BMC-BAMC 2025, Exeter
- 23–4 July LMS-Bath Symposium on Inverse Problems and Artificial Intelligence in Medicine, Bath

July

- 4 LMS General Meeting, London

Calendar of Events

This calendar lists other mathematical events. To promote your event in this calendar, send updates or make corrections, please contact calendar@lms.ac.uk.

February

- 13–15 ICDTHT'25 — The 2025 International Conference on Demographic Transition, Health and Technologies, Salinas, Ecuador

March

- 12–14 33rd Euromicro/IEEE International Conference on Parallel, Distributed Processing (PDP 2025), Turin, Italy

April

- 2–8 Young Functional Analysts' Workshop, Glasgow
- 14–16 Joint Conference of Mathematics Subjects Association — Future Proofing the Curriculum, Loughborough
- 30–2 May 5th IMA and OR Society Conference on Mathematics of Operational Research, Birmingham
- 30–2 May Postgraduate Combinatorics Conference 2025, Glasgow

June

- 23–26 BMC/BAMC 2025 Conference, Exeter
- 23–26 4th IMA Conference on Dense Granular Flows, Cambridge
- 24–26 IMA Mathematics Anxiety International Conference, Cambridge