LONDON MATHEMATICAL SOCIETY
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NEWSLETTER

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SPECTRAL MODELS OF COSMOLOGY

A POSTCARD FROM SEVILLE

NOTES OF A NUMERICAL ANALYST

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Feature content should be submitted to the editor-in-chief at newsletter.editor@lms.ac.uk.

News items should be sent to newsletter@lms.ac.uk.

Notices of events should be prepared using the template at lms.ac.uk/publications/lms-newsletter and sent to calendar@lms.ac.uk.

For advertising rates and guidelines see lms.ac.uk/publications/advertise-in-the-lms-newsletter.

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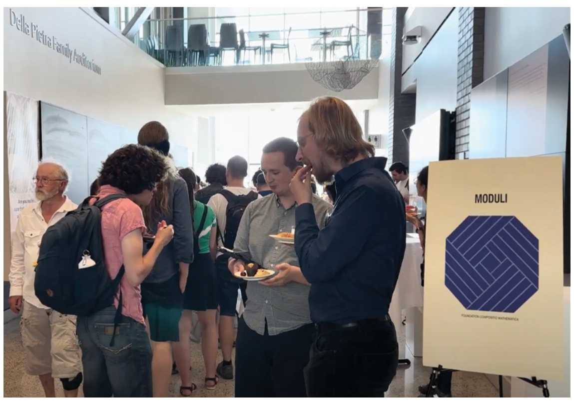
Launch event for new LMS journal *Moduli*

Moduli (cambridge.org/core/journals/moduli), the new open access journal, has been the focus of celebrations at the Simons Center for Geometry and Physics in New York, with presentations of journal papers by contributing authors Claire Voisin, Richard Wentworth, Denis Nesterov and Richard Nabijou. The event formed part of a three-week-long summer workshop on moduli theory.



Participants of the *2nd Simons Math Summer Workshop* celebrating *Moduli*

The following papers are included in the first issue of *Moduli* (2024):

- Arnaud Beauville, Antoine Etesse, Andreas Höring, Jie Liu and Claire Voisin
  Symmetric tensors on the intersection of two quadrics and Lagrangian fibration
- Siqi He, Rafe Mazzeo, Xuesen Na and Richard Wentworth
  The algebraic and analytic compactifications of the Hitchin moduli space
- János Kollár and Sándor Kovács
  KSB stability is automatic in codimension ≥ 3
- Navid Nabijou and Michael Wemyss
  GV and GW invariants via the enhanced movable cone
- Emelie Arvidsson, Fabio Bernasconi and Zsolt Patakfalvi
  On the properness of the moduli space of stable surfaces over $\mathbf{Z}[1/30]$
- Denis Nesterov
  On quasimap invariants of moduli spaces of Higgs bundles
- Renzo Cavalieri, Tyler L. Kelly and Rob Silversmith
  Genus-zero r-spin theory

Moduli is published by the LMS in partnership with Cambridge University Press on behalf of Foundation Compositio Mathematica. Supported by an Editorial Board (moduli.nl/editors) of world-renowned researchers, *Moduli* provides a forum for significant results on all aspects of moduli theory or related mathematics.

Just as moduli theory bridges traditional subject boundaries, the scope of *Moduli* encompasses algebraic, differential and arithmetic geometry, combinatorics, dynamical systems, gauge theory, geometric analysis, geometric group theory, mathematical physics, representation theory and topology.

Moduli is an open access journal, so all articles are free to read. However, the journal does incur real costs for its peer review, typesetting and online distribution. For this reason, the LMS and Cambridge University Press have agreed on a sustainable publishing model under which authors are asked to contribute to the costs of running the journal by taking advantage of funding arrangements for open access publication, where these exist, but no author will be obliged to pay a charge. In most cases, the article processing charge will be covered by publishing agreements already in place between Cambridge University Press and the author's institution.

The Editorial Board is keen to encourage the mathematical community to submit their work to *Moduli*. You can submit your paper at moduli.nl.

LMS President-Elect

The Society is pleased to announce that Professor Mark Chaplain FRSE has been nominated for election as the next President of the LMS.

Professor Chaplain received his PhD in 1990 from the University of Dundee under the supervision of Brian Sleeman. He was appointed as a lecturer in applied mathematics in the School of Mathematical Sciences at the University of Bath in September 1990. In 1996 he moved back to Dundee as a senior lecturer, then reader, before being appointed to a personal chair in mathematical biology in 2000. He was appointed Ivory Chair of Mathematics in 2013 before moving to his current position as Gregory Chair of Applied Mathematics at the University of St Andrews in 2015. While at Dundee he was Head of Division (2006–2013) and was Head of School at St Andrews (2018–2023). He was the recipient of a Leverhulme Personal Research Fellowship (2007-2009) and ERC Advanced Investigator Award (2009–2014).

Professor Chaplain's main area of research is multiscale mathematical modelling of cancer growth and treatment, and he has an international reputation in this field, having carried out pioneering work in this area since the late 1980s. Throughout his research career he has undertaken collaborative, interdisciplinary research with colleagues in biochemistry, developmental biology, cancer biology and clinical oncology. He has developed mathematical models that have provided several new insights into the mechanisms underlying the growth of cancers, and he has laid the foundation for developing novel potential therapeutic treatment strategies via personalised medicine.

He was awarded the LMS Whitehead Prize in 2000 and was elected a Fellow of the Royal Society of Edinburgh in 2003. He has served as a Member-at-Large of the LMS Council (2017–2019). Professor Chaplain has supported the wider mathematical community both nationally and internationally. He has served as Secretary and Treasurer of the European Society for Mathematical and Theoretical Biology (1998–2002), President of the Society for Mathematical Biology (2005–2007) and President of the Edinburgh Mathematical Society (2011–2013).

LMS Prize Winners 2024

The 2024 LMS Prize winners were announced at the Society Meeting on Friday 28 June 2024. We would like to congratulate this year's prize winners for their continued contributions to mathematics.

Professor Gui-Qiang G. Chen of the University of Oxford is awarded the Pólya Prize for his deep research into nonlinear partial differential equations, including his rigorous theoretical analysis of the equations of gas dynamics, especially those involving transonic flows.

Professor Christopher J. Bishop of Stony Brook University, New York, is awarded the Senior Berwick Prize for the pair of papers 'Models for the Eremenko–Lyubich Class', published in the *Journal of the London Mathematical Society* in 2015, and 'Models for the Speiser Class', published in the *Proceedings of the London Mathematical Society* in 2017.

Professor Samir Siksek of the University of Warwick is awarded the Shephard Prize for numerous seductively simple and concrete Diophantine results whose proofs involve a virtuoso display of the most advanced mathematical ideas.

Professor Emmanuel Breuillard of the University of Oxford is awarded the Fröhlich Prize for his landmark work on groups and their actions, masterly combining in ingenious ways algebraic groups, combinatorics, number theory, Diophantine approximation, topology and C*-algebras. His work is notable for its originality, conceptual clarity, elegance and depth.

Dr Ana Ros Camacho of Cardiff University is awarded an Anne Bennett Prize for her ground-breaking work on categorical proofs of the Landau–Ginzburg/conformal field theory correspondence and her tireless dedication to the advancement of women in mathematical physics.

Dr Sabine Bögli of Durham University is awarded a Whitehead Prize for her outstanding contributions to computational spectral theory and to spectral analysis of non-self-adjoint operators, in particular for laying the theoretical foundations for a novel numerical method and for answering several long-standing questions concerning the spectra of non-self-adjoint Schrödinger operators.

Dr Viveka Erlandsson of the University of Bristol is awarded a Whitehead Prize for her outstanding work on curve counting on surfaces. She has also established an extraordinary rigidity theorem for bounce sequences associated with billiard tables. Viveka Erlandsson is a leading figure in low-dimensional geometry, topology and dynamics.

Professor James Newton of the University of Oxford is awarded a Whitehead Prize for his ground-breaking contributions to the Langlands programme and, in particular, for his spectacular joint proof with Jack Thorne of symmetric power functoriality for holomorphic modular forms.

Dr Clarice Poon of the University of Warwick is awarded a Whitehead Prize for her pioneering work at the intersection of optimisation, imaging sciences and machine learning. She has made ground-breaking contributions to the design and analysis of large-scale optimisation schemes aimed at solving ill-posed inverse problems and advancing supervised machine learning techniques. Poon has provided an in-depth mathematical analysis of the super-resolution capabilities of these approaches, showcasing an impressive breadth that covers imaging problems, compressed sensing, neural network training and optimal transport.

Dr Julian Sahasrabudhe of the University of Cambridge is awarded a Whitehead Prize for his outstanding contributions to Ramsey theory, his solutions to famous problems in complex analysis and random matrix theory, and his remarkable progress on sphere packings.

Professor Alessandro Sisto of Heriot-Watt University is awarded a Whitehead Prize for his outstanding contributions to geometric group theory, particularly his ground-breaking work on hierarchically hyperbolic spaces.

Read the full citations at lms.ac.uk/news/2024-lms-prize-winners.

2024 LMS Prize Winners

Gui-Qiang G. Chen
Pólya Prize

Christopher J. Bishop
Senior Berwick Prize

Samir Siksek
Shephard Prize

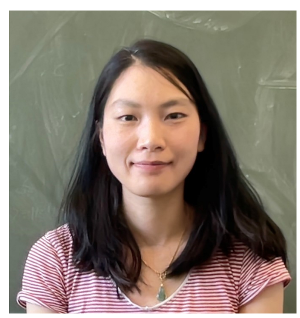
Emmanuel Breuillard
Fröhlich Prize

Ana Ros Camacho
Anne Bennett Prize

Sabine Bögli
Whitehead Prize

Viveka Erlandsson
Whitehead Prize

James Newton
Whitehead Prize



Clarice Poon
Whitehead Prize

Julian Sahasrabudhe
Whitehead Prize

Alessandro Sisto
Whitehead Prize

Louis Bachelier Prize Winner 2024

Congratulations to Professor Peter Tankov, who has been awarded the 2024 LMS/SMAI/Natixis Louis Bachelier Prize.

Peter Tankov is a mathematician with expertise in applied probability and stochastic modelling. He is currently Professor of Quantitative Finance at ENSAE, Institut Polytechnique de Paris (France).

Tankov's research contributions in applied probability, computational finance and mathematical modelling in finance have been influential both in academia and among market practitioners and regulators. He is internationally recognised as a leading academic researcher on green finance, renewable energy markets and the impact of climate risk on financial risk management. His work in these areas, in collaboration with leading climate scientists, economists and regulators, has had significant impact on research and policy.

Read the full citation at tinyurl.com/46xa46j7.

Brady Haran awarded the 2024 Christopher Zeeman Medal

The 2024 LMS/IMA Christopher Zeeman Medal has been awarded to Brady Haran. The Society is delighted to congratulate Brady on this award. The Zeeman Medal is an award in the UK dedicated to recognising excellence in the communication of mathematics. The following citation was provided by Professor Simon Singh.

Brady Haran is the creator of the phenomenally successful YouTube channel *Numberphile* (youtube.com/@numberphile), which features short videos showcasing various levels of mathematics to a worldwide audience. Most videos involve mathematicians being interviewed by Haran, from Fields Medals winners to postgraduate students. Guests have included John Conway, Ron Graham, James Maynard, Timothy Gowers, Terry Tao and the IMA's President, Hannah Fry.

Since 2011, the videos have been watched nearly 700 million times and inspired many young viewers to pursue a career in mathematics (and cause many older viewers to wish they had!). They are among the most viewed mathematics content in the world. While the videos are aimed at a 'lay' audience, they have become a resource for many teachers in schools and universities. Haran also produces the *Numberphile* podcast — extended interviews with mathematicians about their careers.

Haran is also the creator of other extremely popular educational channels. *Computerphile*, which has a strong mathematical element, covers computer science. Other channels focus on chemistry (*Periodic Videos*) and physics (*Sixty Symbols*). He also hosts the *Objectivity* video series, which discusses objects and documents from the archives of the Royal Society and other libraries. Some of the items relate to mathematicians, from Isaac Newton to the amateur mathematician William Shanks. Combined, his videos have been watched over 1.3 billion times and are some of the most watched educational videos on the web.

All the videos are motivated primarily by a desire to share the wonder of the subject but also to show the humanity and personality of the people who work in the various fields.

OTHER NEWS

60th Anniversary of the IMA

The Institute of Mathematics and its Applications celebrates its 60th anniversary in 2024. To mark 60 years of achievement, the IMA has planned an exciting series of events during the year. These look forward to the developments and limitless opportunities for the applications of mathematics in the next 60 years and celebrate the achievements of the past. A history of the origins of the IMA can be found at ima.org.uk/about-us/history/origins-of-the-institute.

LMS Member Recognised in King's Birthday Honours 2024

Congratulations to **Professor Anne Christine Davis OBE** for her award in the 2024 King's Birthday Honours for services to higher education and to scientific research.

Anne is Professor of Mathematical Physics, University of Cambridge. She was a Member-at-Large of the LMS Council from 2020 to 2022 and a member of the Committee for Women and Diversity in Mathematics from 2014 to 2022. She also chaired the Good Practice Scheme Steering Group from 2018 to 2022. This scheme provides support for departments working towards Athena SWAN Award status. In this role, Anne helped to organise and chaired workshops on such topics as the Athena SWAN Charter, support for mathematicians during the pandemic, and EDI and the Research Excellence Framework.

The LMS also congratulates members of the wider mathematical community for their honours:

Paul Fannon OBE, Fellow, Christ's College, Cambridge, and volunteer, United Kingdom Mathematics Trust. For services to education.

Professor Ian Melvyn Hall OBE, Professor of Mathematical Epidemiology and Statistics, University of Manchester and Senior Principal Modeller, UK Health Security Agency. For services to public health, to epidemiology and to adult social care, particularly during covid-19.

Bruno James Michael Reddy OBE, founder and Chief Executive Officer, Maths Circle, Ampthill, Bedfordshire. For services to education.

UK Earns Two Gold Medals at the International Mathematical Olympiad 2024

The International Mathematical Olympiad (IMO) 2024, the 65th IMO, took place on 14–22 July 2024 in Bath, UK, having been originally scheduled for Kyiv, Ukraine. The IMO is the World Championship Mathematics Competition for secondary school students.

The UK team excelled, placing joint sixth out of 108 participating countries. It was the UK's strongest performance since 1995.

The team was made up of six secondary school pupils, who earned two gold medals, three silver medals and one bronze medal. Out of 609 participating students, the UK team achieved the following individual awards:

- Alex Chui: Score 35/42, Gold Medal, 5th out of 609
- Sam Griffiths: Score 28/42, Silver Medal
- Isaac King: Score 25/42, Silver Medal
- Mikaeel Shah: Score 19/42, Bronze Medal

- Samuel Sturge: Score 31/42, Gold Medal, 19th out of 609
- Haolin Zhao: Score 24/42, Silver Medal

Alex Chui, who is now a three-time gold medallist, said:

> I really enjoyed this year's IMO and the preparation together with my teammates and the opportunity to meet contestants from all over the world. I am glad to have got one more gold medal! My favourite question in the contest was the controversial Problem 5 about Turbo the Snail!

Dr Dominic Yeo, King's College London, the UK team leader, said:

> These problems would be found challenging by any mathematician of any age, and we're very proud of the UK team, and especially of Alex and Samuel, for the creativity and breadth of their solutions. The UK's school curriculum is less directed towards this kind of problem-solving than in many other countries, and so the team members have had to work particularly hard to prepare for IMO 2024. This excellent set of results is richly deserved. We hope that the skills they've learned, and the confidence gained from this super performance at IMO 2024, will set them up well for future progress in mathematics and beyond!

The British team was selected and trained by the UK Maths Trust (ukmt.org.uk), a charity that organises mathematics competitions in UK schools. Read more about the IMO at imo-official.org.

OPPORTUNITIES

LMS Education Grants: Call for Applications

Applications are invited for the following grant schemes, which aim to promote interest in mathematics from Key Stage 1 and beyond, and provide support to teachers and other educators:

Small Grants for Education

Grants of up to £800 are available to fund activities that stimulate interest and enable involvement in mathematics from Key Stage 1 (age 5+) to undergraduate level and beyond, by:

- enhancing and enriching mathematical study beyond the curriculum
- engaging the public with mathematics
- encouraging unusual ways of communicating mathematics

See more information at lms.ac.uk/grants/education-grants.

Mathematics Education Conference Grants

This scheme offers up to £2,000 to organisers of regular mathematics education conferences so that they can contribute to the travel or subsistence expenses of the attendees. Note that only one grant may be awarded per conference. See more information at lms.ac.uk/grants/mathematics-education-conference-grants. The deadline for the next round of education grants is 30 November 2024.

LMS Inclusion and Diversity Fund: Call for Applications

The LMS is dedicated to fostering equity and inclusion throughout the mathematics community. This commitment includes providing financial backing to events championing equity, diversity and inclusion (EDI), as well as grant programmes offering direct support to women and under-represented groups in mathematics.

The Society's new Inclusion and Diversity Fund, which is managed by the Committee for Women and Diversity in Mathematics (CWDM), aims to support events that promote EDI in mathematics, broadly speaking. The focus of these events may be on any aspect of diversity, including but not limited to race, ethnicity, gender, trans identity, sexual orientation, sex, age, religion or religious belief, neurodiversity, socio-economic status, disabilities and background. Here are some examples of events that could be funded under this scheme:

- Events that champion the work of people in the mathematical sciences (whether in industry or academia) who come from a particular diversity group or events that offer mentoring and networking opportunities to mathematicians from that diversity group.
- Events that inspire and encourage young people from diverse backgrounds to continue with mathematics.
- Events focused on transforming systems and policies to enhance diversity in mathematics.

The maximum grant that can be awarded per event is £1,000. Applications are now invited. The deadline for submitting applications is 31 October 2024. For details, see:
lms.ac.uk/women/inclusion-diversity-fund.

CWDM offers a number of other grant schemes to support EDI:

- Caring Supplementary Grants: These enable parents and other carers to attend conferences, other research schools, meetings or visits by making a contribution towards their caring costs. The maximum award is £200. Applications are open year-round.
  lms.ac.uk/grants/caring-supplementary-grants
- Grace Chisholm Young Fellowships: These one-year fellowships endorse the holder's status as a mathematician so that a break in their employment does not prevent them from resuming their career at a later stage. The maximum award is £2,000 (£1,500 to the applicant and a £500 stipend to the university). The deadline for the next round is 31 December.
  lms.ac.uk/grants/grace-chisholm-young-fellowships
- Emmy Noether Fellowships: These one-year fellowships enhance the maths research of holders who are either re-establishing their career after a break or dealing with significant caring responsibilities. Awards are for between £2,000 and £10,000. The next round will open in early 2025.
  lms.ac.uk/grants/lms-emmy-noether-fellowships

LMS Research Grants: Call for Applications

Research Grant Applications

The next closing date for research grant applications (Schemes 1 to 5 and Mathematics in Africa) is 15 September 2024. Applications are invited for the following grants to be considered by the Research Grants Committee at its October 2024 meeting. Applicants for LMS grants should be mathematicians based in the UK, the Isle of Man or the Channel Islands. For grants that support conferences or workshops, the event must be held in the UK, the Isle of Man or the Channel Islands.

Conferences and Workshops (Scheme 1)

Please note that as of 1 August 2024, the LMS Scheme 6 Workshop Grant has been merged with the Scheme 1 Conference Grant to become the Scheme 1 Conference and Workshop Grant.

Grants of up to £5,500 are available to provide partial support for conferences and workshops. This includes travel, accommodation and subsistence expenses for principal speakers, UK-based research students, participants from Scheme 5 countries and caring costs for attendees who have dependants.

Visits to the UK (Scheme 2)

Grants of up to £1,500 are available to provide partial support for a visitor who will give lectures in at least three separate institutions. Awards are made to the host for the travel, accommodation or subsistence costs of the visitor. Potential applicants should note that it is expected that the host institutions will contribute to the expenses incurred by the visitor. In addition, the Society can offer a further amount of up to £200 to cover the caring costs for those who have dependants.

Research in Pairs (Scheme 4)

These grants are available to mathematicians inviting a collaborator to their home base or visiting a collaborator at another institution.

Grants of up to £1,200 are available to support a visit for collaborative research either by the grant holder to another institution abroad or by a named mathematician from abroad to the home base of the grant holder. For mathematicians collaborating with another UK-based mathematician, grants of up to £600 are available to support a visit for collaborative research either by the grant holder to another institution or by a named mathematician to the home base of the grant holder. In addition, the Society can offer a further amount of up to £200 to cover the caring costs for those who have dependants.

Research Reboot (Scheme 4)

Grants of up to £500 for accommodation, subsistence or travel plus an additional £500 for caring costs are available to assist UK mathematicians who may have found themselves with very little time for research due to illness, caring responsibilities, increased teaching or administrative loads, or other factors. The aim of this scheme is to allow the researcher to leave their usual environment so that they can focus entirely on their research for a period from two days to a week. For applications submitted by the next deadline (22 January 2024), the reboot retreats should take place between 15 March and 30 June 2024.

Collaborations with Developing Countries (Scheme 5)

For mathematicians inviting a collaborator to the UK, grants of up to £3,000 are available to support a visit for collaborative research by a named mathematician from a country in which mathematics could be considered to be in a disadvantaged position to the home base of the grant holder. For mathematicians going to their collaborator's institution, grants of up to £2,000 are available to support a visit for collaborative research by the grant holder to a country in which mathematics could be considered to be in a disadvantaged position. Applicants will be expected to explain in their application why the proposed country fits the circumstances considered eligible for Scheme 5 funding. In addition, the Society can offer a further amount of up to £200 to cover the caring costs for those who have dependants. Contact the Grants team if you are unsure whether the proposed country is eligible or check the definition of a developing country adopted by the IMU's Commission for Developing Countries (tinyurl.com/y9dw364o).

Mathematics in Africa Grants

Grants of up to £2,000 are available to provide partial support for mathematical activities based in Africa. Please contact grants@lms.ac.uk for more information.

Early Career Research Grants

The next closing date for early career research grant applications (Schemes 8 and 9 and ECR travel grants) is 15 October 2024. Applications are invited for the following grants to be considered by the Early Career Research Committee at its November 2024 meeting.

Postgraduate Research Conferences (Scheme 8)

Grants of up to £2,500 are available to provide partial support for conferences organised by and are for postgraduate research students. The grant award is to be used to cover the costs of the participants. In addition, the Society allows the use of the grant to cover the caring costs for those who have dependants.

Celebrating New Appointments (Scheme 9)

Grants of up to £400 or £500 are available to provide partial support for meetings to celebrate the new appointment of a lecturer at a university. Potential applicants should note that it is expected that the grant holder will be one of the speakers at the meeting. In addition, the Society allows the use of the grant to cover the caring costs for those who have dependants.

ECR Travel Grants

Grants of up £500 are available to provide partial support for travel or accommodation to allow UK-based early career researchers to attend conferences or undertake research visits either in the UK or overseas.

Atiyah UK—Lebanon Fellowships

The LMS is pleased to announce the opening of a new round of applications for the Atiyah UK–Lebanon Fellowships 2024/2025. It invites applications from established UK-based mathematicians who wish to visit Lebanon and from mathematicians from Lebanon who are at the level of an advanced MSc or PhD student or above and who wish to visit the UK.

Since the scheme was launched in 2020 to honour the memory of Sir Michael Atiyah (1929–2019), the Atiyah UK–Lebanon Fellowships have been fostering exchange and interaction between mathematicians in the two countries by supporting a two-way visiting programme. The scheme operates in partnership with the Centre for Advanced Mathematical Sciences (aub.edu.lb/cams/Pages/default.aspx) at the American University of Beirut.

Professor Ali Wehbe

In 2023/2024, the Atiyah UK–Lebanon Fellowships Panel awarded a fellowship to Professor Ali Wehbe at the Lebanese American University and head of Al-Khawarizmi Laboratory of Mathematics and its Applications. Professor Wehbe has authored more than 67 original papers that have been published in international journals, covering three primary research domains: (1) the asymptotic behaviour of physical systems governed by partial differential equations, (2) optimal control, which has applications in both physical sciences and medicine, and (3) the numerical analysis of distributed systems. The funding provided by the Atiyah UK–Lebanon Fellowship will support Professor Wehbe's visit to Newcastle University from July to September 2025, during which time he will work with Professor David Seifert and other members of the faculty on mathematical models of the interface between different types of material, especially those with a fractional derivative in the boundary conditions.

If an Atiyah UK–Lebanon Fellowship could benefit your career, please apply online by 31 January 2025. Further information about Atiyah UK–Lebanon Fellowships for the academic year 2025/2026 can be found on our website (lms.ac.uk/grants/atiyah-uk-lebanon-fellowships).

LMS Early Career Fellowships 2024/2025

With support from the Heilbronn Institute for Mathematical Research and UKRI

An LMS Early Career Fellowship supports early career mathematicians in the transition between PhD and a postdoctoral position. The LMS offers fellowships of between 3 and 6 months to mathematicians who have recently or will shortly receive their PhD. The award will be calculated at £1,615 per month plus an £800 allowance for relocation or collaboration visits.

Fellows must pursue strong research links outside the institution where they received their PhD. This can be done, for example, by changing institutions or by making collaborative visits during the fellowship.

A fellowship may be held at one or more institutions.

These fellowships are supported by Heilbronn Institute for Mathematical Research through the UKRI/EPSRC Additional Funding Programme for Mathematical Sciences.

The application deadline is 13 January 2025.

For full details of this scheme plus information on eligibility criteria and how to apply, please visit the LMS website (lms.ac.ul/grants/lms-early-career-fellowships).

Queries regarding applications can be addressed direct to Nicola Goldie, Committee, Grants and Membership Manager (fellowships@lms.ac.uk).

Cecil King Travel Scholarships: Call for Applications

The LMS administers two £6,000 travel awards funded by the Cecil King Memorial Foundation for early career mathematicians. These can be used to support a period of study or research abroad, typically of three months. One scholarship will be awarded to a mathematician in any area of mathematics and one to a mathematician whose research is applied in a discipline other than mathematics.

The application deadline is 15 November 2024.

For full details of this scheme, information on eligibility criteria and how to apply, visit the LMS website (lms.ac.uk/prizes/cecil-king-travel-scholarship).

Queries regarding applications can be addressed direct to Nicola Goldie, Committee, Grants and Membership Manager (fellowships@lms.ac.uk).

Pre-registration of interest

Heilbronn Postdoctoral Research Fellowships

Are you interested in using your skills in pure mathematics, data science or quantum information as part of a team working on exciting real world mathematical problems that help to keep the UK safe? Our **3 year Heilbronn Postdoctoral Research Fellowships** provide the opportunity to continue your own personal research alongside working on varied and fascinating classified research projects, collaborating with colleagues in a supportive and encouraging environment that puts an emphasis on teamwork. Research areas of interest include, but are not restricted to, Algebra, Algebraic Geometry, Combinatorics, Data Science, Number Theory, Probability, and Quantum Information. Fellows have previously been appointed with backgrounds in most areas of Pure Mathematics, Data Science and Statistics, Quantum Information and Mathematical/Theoretical Physics. You will also be offered training and development opportunities that fit your career ambitions, research needs and personal development requirements.

You are invited to complete this form to pre-register your interest in our **Fellowship positions**. Our full job advert will go live in November and we expect to make a number of appointments at the University of Bristol, Imperial College London, King's College London, University College London, and the University of Manchester for October 2025. Heilbronn Fellows divide their time equally between their own research and the classified research programme of the Heilbronn Institute.

Starting salary: £45,232-£50,474 (or local equivalent) depending on previous experience, plus the London weighting where appropriate. In addition, a fund of £2.5K pa to pay for research expenses will be available to each Fellow. Our Fellowships are **3 years fixed term, full-time**, with a preferred start date of 1 October 2025.

Equal Opportunities - Diversity and inclusion are critical to the vision and mission of the Heilbronn Institute. We seek a truly diverse workforce which includes diversity in every sense of the word: those with different backgrounds, ages, ethnicities, gender identities, sexual orientations, ways of thinking and those with disabilities or neurodivergent conditions. We therefore welcome and encourage applications from everyone, including those from groups that are currently under-represented in mathematics.

Due to the nature of the Institute's work, Fellows must be willing to apply for a national security clearance at an appropriate level. UK resident UK nationals will normally be able to meet this condition: other potential applicants are welcome to contact the Heilbronn Manager at himr-recruitment@bristol.ac.uk to discuss eligibility.

More information about the Institute may be found at our website https://heilbronn.ac.uk

Pre-registration link - https://forms.office.com/e/vvf3BgnA2Y

Shape the Future of the LMS by Voting in the Society's Elections 2024

Engaging with its members through ballots helps to keep the Society's governing body fresh, accountable and credible while bringing in diverse opinions and enabling members to connect with the Society's Council.

The Society promotes fair and transparent elections that contribute to the general welfare of the mathematical community and portray a positive public image of its members. Competitive elections allow the Society to form a strong governing body, which is responsible for the general control and management of its administration, strategies, plans and the charity's financial operations.

Voting in the annual Council and Nominating Committee elections, which will take place in October and November 2024, is an excellent way to get involved and influence the future of the Society. Voting will open on Friday, 18 October 2024. Instructions on how to vote will be sent to members by email or post by Civica Election Services, the organisation that is administering the elections. Members are encouraged to check that their contact details are up to date at lms.ac.uk/user.

The results of the Council and the Nominating Committee elections will be announced at the Annual General Meeting, which will be held on Friday, 22 November 2024.

Please visit our website for more information (lms.ac.uk/about/council/lms-elections).

Update from the Council Regarding an Official LMS Email Address for Members

At the LMS Council Strategic Retreat in April 2023, there was a proposal to revisit the idea of offering official LMS email addresses to retired members. It was agreed at the LMS Council in February 2024 to complete a survey of the membership to find out more about the potential interest and whether this could be a chargeable benefit. At its meeting on Friday, 19 April 2024, the LMS Council had a discussion on the responses and feedback from members regarding this idea. After careful consideration, the Council agreed not to look at this any further due to a lack of interest.

LMS Membership and Its Benefits

Founded in 1865, the LMS stands as one of the most prestigious mathematical societies in the world with its vibrant community of over 3,000 members from around the globe. Its mission to enable, advance, disseminate and promote mathematical knowledge can be accomplished only through active engagement with its members. The LMS is also committed to fostering inclusivity and diversity within the mathematical community. Through targeted grant programs, the Society supports initiatives aimed at increasing participation among under-represented groups in mathematics. Such efforts are essential for creating a more equitable and representative mathematical community. Being a part of such a community offers numerous advantages and benefits.

Free Access to High-Quality Publications

The LMS publishes several renowned journals, including the *Bulletin of the LMS*, the *Journal of the LMS* and the *Proceedings of the LMS*, and offers free online access to more of such first-class mathematical content. Continued membership ensures uninterrupted access to these leading publications and allows members to keep up with the latest innovations and research findings in the field. These resources are invaluable for academics, researchers and practitioners who rely on current knowledge to inform their work and foster innovation.

Each member also receives regular members-only LMS e-Updates and quarterly newsletters to keep them up-to-date with the latest mathematical developments, including policy issues, details of forthcoming events, book reviews and more. Further information can be found on our website (lms.ac.uk/publications).

Engagement in a Thriving Mathematical Community

Membership in the LMS is more than just access to resources; it means belonging to a vibrant and dynamic community of mathematicians. Continued connection with this community fosters a sense of belonging and shared purpose. The Society acts as a forum for intellectual exchange, mutual support and celebrating mathematical achievements. This sense of community is invaluable for personal and professional fulfilment.

Networking and Professional Development

The LMS provides great opportunities for networking with peers, mentors and leading figures in mathematics. Regular events, such as conferences, workshops and seminars, offer members the opportunity to engage with leading research, share their work and collaborate with others in the community. These interactions are not only vital for professional growth but also for building relationships that can lead to collaborative projects and career advancement. For a full list of events, please see our website (lms.ac.uk/events).

Advocacy and Support for Mathematics

The LMS plays a pivotal role in advocating for the interests of mathematicians and the broader mathematical sciences community. By continuing with their membership, individuals support the Society's efforts to influence policy, secure funding for research and promote the importance of mathematics in education and society. This collective voice is crucial in ensuring that mathematics receives the recognition and resources it deserves.

To see a full list of membership benefits please visit our website (lms.ac.uk/membership/member-benefits).

Renewing Your Membership for 2024/2025

All current members of the Society are now invited to renew their membership for the period from November 2024 to October 2025 so that they continue to support our mission to disseminate and promote mathematics by encouraging mathematical research, education and communication. Each member will receive in September an update about the new subscription rates as well as guidance on how to update their information, renew their subscription and subscribe to the LMS publications. A full list of membership rates and instructions on how to renew your subscription are available on our website (lms.ac.uk/membership/paying-your-subscription).

How to Join

Whether you are a student, a teacher, a researcher or simply someone who loves mathematics, we welcome you to join our community. To become a member, please complete the online application form by 18 November 2024 so that you can be elected at the Annual General Meeting on 22 November 2024. All applications need to be proposed and seconded by members of the Society. The LMS has representatives at many universities in the UK who can endorse your application. Any applicant who has difficulty in finding a proposer or seconder should contact the LMS Membership Department.

For more information on how to join, please visit our website (lms.ac.uk/membership/how-join).

If you have any queries, please contact the LMS membership team via membership@lms.ac.uk or call us on 020 7927 0808 or 020 7291 9973.

LMS Council Diary — A Personal View

A hybrid meeting of the Council took place on Friday, 19 April 2024. The President Jens Marklof started by reporting on several activities he had engaged with since the last Council meeting in February 2024. Notably, the UK's first *Maths Summit*, held at the Science Museum on 12 March, was a resounding success and attracted substantial media coverage. The event brought together leaders from academia, industry, education and politics to explore how the mathematical sciences can support research, innovation and prosperity. A Maths Manifesto, produced under the leadership of LMS Vice-President Cathy Hobbs and launched at the *Maths Summit*, was commended. The *Summit* was organised by Protect Pure Maths (since renamed the Campaign for Mathematical Sciences), a campaign to protect pure mathematics and advance the mathematical sciences in the UK. The President reiterated the need to continue to engage proactively with those institutions where mathematics degrees are under considerable strain.

The Council was then updated on the work of the LMS Academy for the Mathematical Sciences working group and the most recent steps taken towards the creation of a National Academy. The Council for Mathematical Sciences (CMS) had formulated a response to the UK Government DSIT consultation launched in early 2024, which had been endorsed by all CMS members and the five learned societies. The LMS had taken this opportunity to highlight key points from its own perspective. DSIT subsequently published their response to the consultation. The CMS and the proto-Academy have sent a short supportive response to this publication to DSIT. The Council reiterated that both the use and discovery of new mathematics should be advocated. Regarding the next concrete steps, the Council was informed that the role of the President of the Academy had been advertised (Professor Alison Etheridge has now been appointed) and that the Fellowship of the Academy model was taking shape. Further details on this and more were shared with the Council by three members of the Executive Committee of the proto-Academy — Ruth Kaufman (Lead for Governance and Fellowship), Nigel Campbell (Chair) and Tom Coates (LMS Academy Liaison) — in an online meeting at the beginning of June.

Other highlights of the meeting included: (1) A discussion on how the LMS might engage more with honorary members. Several ideas were aired but the topic will be revisited in the future. (2) A discussion on how to encourage more individuals with a keen interest in mathematics to become members. (3) A presentation of the results of a survey on the interest of members to have an email account linked to the LMS. Given the low number of positive responses, this idea has been abandoned.

Two items of committee business caught my attention. First, the Publications Committee has negotiated and signed eleven agreements, variations and letters of intent in relation to the publishing portfolio. All journals, except for *Compositio Mathematica*, are now published online only. The Committee has also restructured the shared editorial board of the *Bulletin of the London Mathematical Society* and the *Journal of the London Mathematical Society* so that these journals now welcome specialised papers with a well-written introduction. It is anticipated that these changes will enable the journals to publish more articles without compromising the quality of the research published. The Council was reminded that publishing with the LMS contributes to putting money back into mathematics through initiatives taken by the LMS.

Second, the Prize Committee asked the Council to consider the 2024 list of LMS Prize Winners and the corresponding citations. The Committee also submitted a paper to the Council regarding revised guidelines for the LMS Prizes. In particular it was approved that, given the disproportionately low number of women and other under-represented groups nominated for prizes each year, prize regulations would replace age restrictions with the concept of 'academic age' to take account more fully of broken career patterns.

The Prize Winners were publicly announced at the General Meeting of 28 June 2024, just after the subsequent Council meeting. (A list of winners can be found on page 5 in this issue.)

Anne Taormina
Member-at-Large of Council

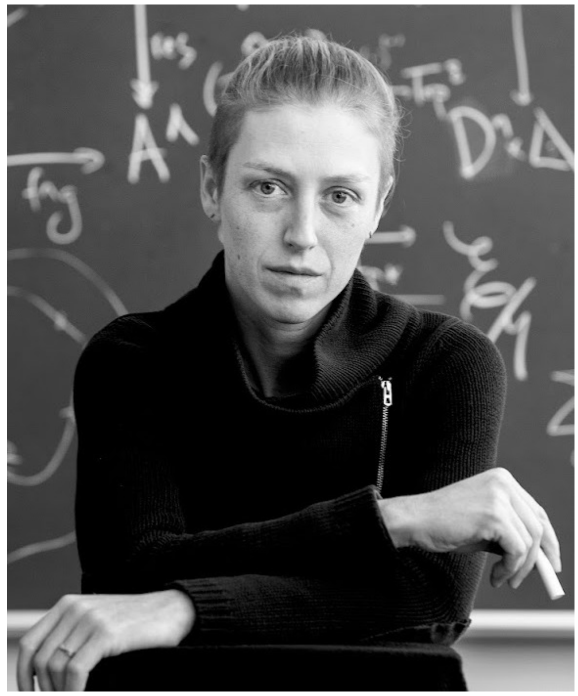
LMS Hardy Lectureship 2025

The LMS is pleased to announce the LMS Hardy Lecturer 2025 is Professor Emily Riehl (Johns Hopkins University).

Professor Riehl has established herself as a leading expert in higher category theory and has also developed an interest in connections with computer science such as homotopy type theory. She is an accomplished and enthusiastic expositor of mathematics at a variety of levels aimed at mathematicians and the wider public, having had articles published in *Scientific American* and *New Scientist*. She also plays a leading role in the broader engagement of mathematicians and other scientists from marginalised and discriminated-against groups.

Professor Riehl will undertake a lecture tour of the UK in June and July 2025, which will include the Hardy Lecture at the Society Meeting on Friday, 4 July, in London.

The Hardy Lectureship was founded in 1967 in memory of G.H. Hardy to recognise his outstanding contribution to both mathematics and to the Society. The Hardy Lectureship is a lecture tour of the UK by a mathematician with a high reputation in research.



Professor Emily Riehl

Further details about the Hardy Lecture Tour 2025 are available on the website here: lms.ac.uk/events/lectures/hardy-lectureship.

REPORTS OF THE LMS

Report: LMS Mathematics Communications Workshop, May 2024, University of Manchester

The LMS has been running communications and outreach training days for mathematicians at all levels for the last 5 years. The aim is to develop a cadre of enthusiastic communicators of mathematics in the UK in support of its mission to promote mathematical knowledge, including to young people and the wider public.

On a sunny Tuesday in early May, 15 mathematicians from master's level to professor gathered in Manchester for an energising day thinking and practising mathematics communication with experienced and expert mathematics communications professionals Dr Katie Steckles and Dr Ben Sparks.

Activities ranged from summarising a mathematical topic into 100 words (a good discipline — try this for your favourite topic!), pitching a section of your next mathematics communication event for critique from Katie, Ben and other participants, and experimenting with physical objects such as Penrose tiles and business cards that can be used to get mathematical ideas across to non-specialists. We were introduced to the basics of communication — knowing your audience, your aims and the resources available — as well as aspects specific to mathematics communication. Above all, enthusiasm for your subject is key, as ably demonstrated by Katie and Ben throughout the day.

We expect to hold the next Maths Communications Workshop in spring 2025.

Professor Cathy Hobbs
LMS Vice-President

Report: LMS Education Day 2024

The 2024 LMS Education Day took place at De Morgan House on 24 May. The event highlighted the challenges and opportunities that digital technologies and AI tools present in supporting the teaching and learning of mathematics. The event was opened by the chair of the LMS Education Committee, Professor Mary McAlinden (Nottingham Trent University), who was followed by a keynote presentation from Professor Michael Grove (Birmingham University) on 'Generative AI: Its implications, and opportunities, for policy, pedagogy and practice within the mathematical sciences.' This talk generated a lively Q&A session and much discussion that extended well into lunchtime. Lunch also provided an excellent networking opportunity, as attendees discussed the implications of incorporating or not incorporating these new tools into their teaching and research practices.

After lunch, the afternoon began with a short session covering three recent discussion papers and reports on the use of digital technology in mathematics at various levels of education. First, Dr Sofya Lyakhova (Swansea University) presented a discussion paper that she had produced for the JMC entitled 'On the use of technology in university mathematics teaching and assessment in STEM degree schemes' (JMC Digital Technologies and Mathematics Education Working Group discussion paper, June 2023). This paper included four case studies showcasing various uses of technology in university teaching.

Next, Dr Cosette Crisan and Dr Eirini Geraniou (UCL Institute of Education) presented the report 'Educational technologies in mathematics education,' which had been commissioned by the Royal Society. This report, produced in collaboration with Professor Jeremy Hodgen (Royal Society Mathematical Futures Programme Project report, April 2023), offered recommendations to help educators identify the best digital technologies for different purposes in mainstream mathematics education, including at primary and secondary school levels as well as higher education. They suggested strategies for successfully integrating these technologies.

Finally, Professor Alison Clark-Wilson (UCL Knowledge Lab) introduced a report from a JMC working group that she had co-edited with Professor Andrew Noyes, Professor Jeremy Hodgen and Tom Button. The report, 'Mathematics education and digital technology' (JMC report, July 2023), revisited the 2011 JMC report on 'Digital technologies in mathematics education' and provided an update on the progress made against its recommendations.

The final session of the afternoon was a carousel of three interactive session with attendees having the opportunity to attend two out of the three:

- Dynamic Digital Tools in Teaching and Learning
  This session was delivered by Tom Button (MEI) and introduced delegates to dynamic maths tools, GeoGebra and Desmos in particular. Attendees were informed about the phone apps for both tools, highlighting their powerful yet straightforward usability. The session covered the main functionalities of each digital tool, enabling attendees to make informed decisions about which to incorporate into their practice.

- Digital Marking Tools
  In this session, Dr Beatrice Navarro Lameda (UCL) provided a guided tour of Crowdmark and shared her experiences with the software. Attendees had the opportunity to gain hands-on experience with Crowdmark. This was followed by Professor Paul Glaister (University of Reading), who contrasted his experience of using Gradescope as a tool for online marking.

- Computers and Theorem Proving (LEAN)
  In this session, Dr Athina Thoma (University of Southampton) and Dr Gihan Marasingha (Exeter University) gave a live demonstration of LEAN, an interactive theorem prover that is gaining interest as a tool for teaching pure mathematics to undergraduate students. Through hands-on activities, they discussed the potential of LEAN in supporting the transition to proof and logic modules.

To conclude, the 2024 LMS Education Day effectively showcased the potential of digital technologies and AI tools in mathematics education and facilitated rich discussions and hands-on experiences for the 60+ attendees, who gained valuable insights into the evolving landscape of mathematics teaching and learning.

Christopher Saker and Cosette Crisan
LMS Education Committee

Report: LMS Teaching and Learning in a HE Workshop – Threshold Concepts in Undergraduate Mathematics

A UK-wide one-day workshop on identifying and recognising the implications of threshold concepts in the teaching of undergraduate mathematics was hosted by Dr Rehan Shah on Thursday, 30 May 2024, at Queen Mary University of London, which was funded through an LMS HE Teaching and Learning Grant.

The workshop, consisted of four sessions that involved a mix of theoretical discussions and practical, interactive work led by Dr Rehan Shah (Queen Mary University of London) and his colleagues Dr Manish Malik (Canterbury Christ Church University), Professor Ann O'Shea (Maynooth University), Dr Sinead Breen (Dublin City University) and Dr Matheus Oliveira De Andrade (University College London).

The first session, led by Professor O'Shea and Dr Breen and titled 'Introduction to threshold concepts,' provided participants with a thorough background of threshold concepts and their characteristics. This was followed by an interactive Mentimeter element, during which the audience could share their thoughts on some examples cited within the prior literature.

The second session, which was led by Dr Shah and titled 'Identifying and recognising threshold concepts in undergraduate mathematics,' was an engaging and interactive one in which participants worked together in groups to apply their knowledge from the previous session to identify and classify potential threshold concepts from a description of a sample mathematics module.

The third session, helmed by Dr Andrade and titled 'Threshold concepts: Implications on curriculum design,' provided participants with the theoretical notions of knowledge semantics, epistemic and social relations, hidden curriculum and semantic gravity in the context of some practical examples of mathematics topics.

The final interactive session was led by Dr Malik. The participants were asked to apply their knowledge from the three previous sessions to design formative multiple-choice questions computationally by making use of generative AI using ChatGPT with the help of carefully designed prompts.

The workshop was an overwhelming success and received extremely positive feedback from its 40 academic and educator participants, who were from a diverse range of national and international universities, including some from Ireland, Iceland and India. Feedback comments received from some of the attendees included: "What a fantastic workshop. The day was structured so well – it allowed us to get a state of theory, practice and discussion in alternating sessions which made what we learnt very fresh and interesting."

I had great opportunities to network with my peers and met some amazing people, all of whom were very friendly and interested in further discussions.

The whole workshop was very useful and well organised and planned, alternating between theoretical and practical sessions. The sessions were highly interactive, thought-provoking and insightful. They gave us an opportunity to think deeply about the characteristics of threshold concepts.

The workshop will be used to develop a community of practice among all those who attended, and the findings will also help to inform further research studies in this area. We are all deeply grateful to the support provided by the LMS Education Committee, which enabled us to run this workshop. If anyone is interested in having us deliver a similar workshop or seminar at their university, please do get in touch (rehan.shah@qmul.ac.uk).

Rehan Shah
Queen Mary University of London

Records of Proceedings at LMS meetings
General Meeting & Celebration of Kelvin's 200th Anniversary 2024 (in partnership with BSHM and University of Glasgow)

A General Meeting of the LMS was held on 28 June 2024 at De Morgan House, 57–58 Russell Square, London. Over 80 members and guests were present for all or part of the meeting, either in person or online.

The meeting began at 10.30am with a warm welcome from representatives of the LMS, The British Society for the History of Mathematics and Glasgow University.

The first lecture was given by Mark McCartney (Ulster University) and titled *Ever in Haste*.

The second lecture was given by Luke K. Davis (University College London), entitled *If You Cannot Measure It, You Cannot Control It: Active Matter Under Thermodynamic Control*.

After lunch, Ruiping Mu (Northwest University) gave her lecture, entitled *Thomson and Stokes' Work on the Establishment of the Dynamical Equation and Its Associated Proofs*, which was followed by a lecture from Rosalba Garcia-Millan (Kings College London) entitled *Non-equilibrium Thermodynamics and Field Theory in Active Matter*.

After a tea break, the President, Professor Jens Marklof, opened the LMS Annual General Meeting, during which members were asked to vote on four items of business.

(1) Minutes of the Annual General Meeting held on 17 November 2023
The minutes of the Annual General Meeting held on 17 November 2023 were agreed as an accurate record.

(2) Appointment of scrutineers for 2024
The Council proposed that Professor Charles Goldie and Professor Cho-Ho Chu be appointed as scrutineers for the 2024 LMS elections for Council and Nominating Committee.

The proposal was carried unanimously.

(3) Appointment of auditors for 2024
The Council proposed that Griffin Stone Moscrop and Company be appointed as the Society's auditors for the current financial year, 2023/2024. Following a tender exercise, the President recommended on behalf of the Council the approval of the appointment of Griffin Stone Moscrop and Company as new auditors to the Society and asked members if they agreed.

The resolution required a simple majority to be passed and was passed.

(4) Honorary Memberships
The Council proposed three Honorary Members be elected in 2024: Professor Sylvia Bozeman, Professor Grigory Margulis and Professor Shigefumi Mori. The President read brief citations for each.

It was unanimously agreed to elect Professor Sylvia Bozeman, Professor Grigory Margulis and Professor Shigefumi Mori as Honorary Members of the Society.

The Annual General Meeting continued by electing 26 members to Membership of the Society. Three members signed the Members' Book.

The President then announced the winners of LMS Prizes in 2024.

Pólya Prize	Professor Gui-Qiang G. Chen
Senior Berwick Prize	Professor Christopher J Bishop
Shephard Prize	Professor Samir Siksek
Fröhlich Prize	Professor Emmanuel Breuillard
Anne Bennett Prize	Dr Ana Ros Camacho
Whitehead Prizes	Dr Sabine Bögli
	Dr Viveka Erlandsson
	Professor James Newton
	Dr Clarice Poon
	Dr Julian Sahasrabudhe
	Professor Alessandro Sisto

The President also announced the winner of the LMS/SMAI/Natixis Louis Bachelier Prize winner in 2024, Professor Peter Tankov. The President offered his congratulations to the prize winners on behalf of the Society.

The President then closed the LMS Annual General Meeting and thanked all the organisers of the Celebration of Kelvin's 200th Anniversary meeting and everyone attending.

The Anniversary celebration then continued with a lecture from Jemma Lorenat (Pitzer College), entitled *An Illustrated History of Drawing Knots*.

To finish, Joe Goddard (UC San Diego) gave a lecture entitled *William Thomson and Thermoelectricity*.

Stephen Watson (Glasgow) closed the meeting at 6.15pm with a round-up of the day's events and thanked all the speakers for their excellent lectures.

A wine reception was held at De Morgan House following this event, and the Society dinner was then held at Antalya, 103–105 Southampton Row, London.

Records of Proceedings at LMS meetings
Ordinary Meeting: LMS Spitalfields History of Mathematics Meeting and Hirst Lecture 2024

This meeting was held on Friday, 26 April 2024, in person at De Morgan House and virtually via Zoom. Over 30 members and guests were present in the Hardy Room, and over 20 members and guests were present on Zoom, for all or part of the meeting.

The meeting began at 2.30pm, with the President, Professor Jens Marklof, in the Chair. This was Professor Marklof's first time chairing an ordinary meeting, since the beginning of his term.

Several members signed the Member's Book and were admitted to the Society.

The Chair introduced the first talk, which was given in person by Jeremy Gray (The Open University) on *F.S. Macaulay and Modern Commutative Algebra*.

After the break, the Chair introduced the second talk given in person by the winner of the LMS-BSHM Hirst prize, Erhard Scholz (Bergische Universität Wuppertal), on *From Grassmann Complements to Hodge Duality*.

Upon the lectures' conclusion, Professor Marklof thanked the speakers for their excellent lectures and then expressed the thanks of the Society to the organisers, the SLAM Committee and the BSHM for a wonderful meeting and workshop.

Afterwards, a wine reception was held at De Morgan House.

UNIVERSITY OF CAMBRIDGE

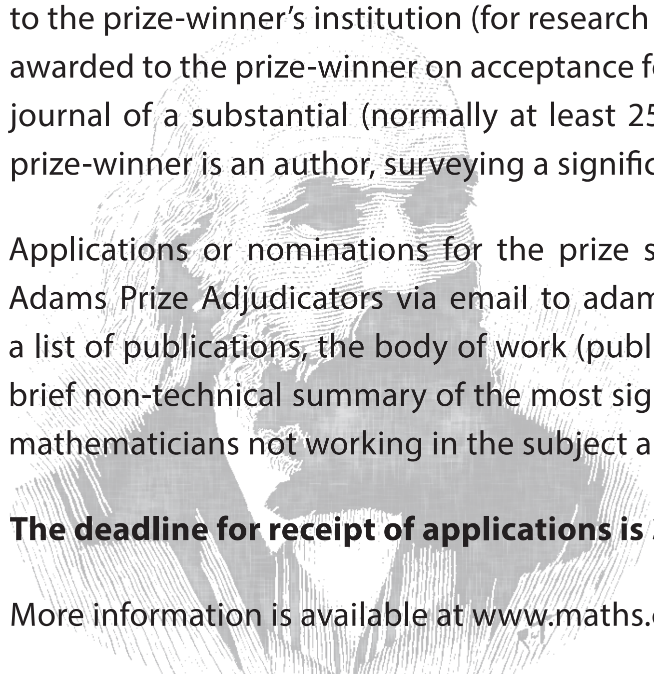
ADAMS PRIZE

Mathematics of Statistical Mechanics

The University of Cambridge has announced the subject for one of its oldest and most prestigious prizes. The Adams Prize is named after the mathematician John Couch Adams and was endowed by members of St John's College. It commemorates Adams's role in the discovery of the planet Neptune, through calculation of the discrepancies in the orbit of Uranus.

The Chair of the Adjudicators for the Adams Prize invites applications for the 2024-2025 prize which will be awarded this year for achievements in the field of Mathematics of Statistical Mechanics.

The prize is open to any person who, on 31st October 2024, will hold an appointment in the UK, either in a university or in some other institution; and who is under 40 (the Adjudicators may vary this age limit, on a case-by-case basis, where they consider it fair and reasonable to do so. Applicants may consult the Adjudicators in advance for a determination of eligibility). The value of the prize is expected to be approximately £30,000, of which one third is awarded to the prize-winner on announcement of the prize, one third is provided to the prize-winner's institution (for research expenses of the prize-winner) and one third is awarded to the prize-winner on acceptance for publication in an internationally recognised journal of a substantial (normally at least 25 printed pages) original article, of which the prize-winner is an author, surveying a significant part of the winner's field.



Applications or nominations for the prize should be submitted to the Secretary of the Adams Prize Adjudicators via email to adamsprize@maths.cam.ac.uk together with a CV, a list of publications, the body of work (published or unpublished) to be considered and a brief non-technical summary of the most significant new results of this work (designed for mathematicians not working in the subject area).

The deadline for receipt of applications is 31st October 2024.

More information is available at www.maths.cam.ac.uk/adamsprize

Spectral Models of Cosmology

MATILDE MARCOLLI

Modified gravity models

The large supply of cosmological data provided by the latest generation of satellites and telescopes, including the Planck map of the cosmic microwave background radiation (CMB), has led to a corresponding broad development of cosmological models that attempt to capture possible deviations at very large scales and in the very early universe from the setting of general relativity and the standard cosmological model. In particular, questions related to the expansion of the universe, and the related mysteries of dark matter and dark energy, may have an answer in terms of yet unknown elementary particles or else, perhaps, in terms of physically acceptable modifications to the laws of gravity.

In this context, several possible models have been developed and analysed, generally labelled as 'modified gravity models.' A class of such models includes modifications of the Einstein–Hilbert action functional of general relativity that have functions $f(R)$ of the scalar curvature. Other gravity action functionals include conformal gravity, which is based on the Weyl curvature tensor and a possible pairing of the scalar curvature to a scalar field. This model has seen various possible cosmological applications, from the Hoyle–Narlikar cosmologies, proposed as alternatives to the standard Friedmann-Robertson-Walker space-times of the hot Big Bang model, to more recent proposals. Other higher-derivative modifications of the gravity action functional include Gauss–Bonnet gravity, Lovelock gravity, which includes higher-order contractions of the Riemann curvature tensor, as well as bimetric theories, scalar-tensor theories, MOND, etc.

This wealth of competing modifications of the Einstein–Hilbert action and their possible cosmological implications have been a central object of study in contemporary theoretical cosmology but have so far attracted little attention from the mathematical community.

I am going to illustrate here how techniques developed within the context of non-commutative geometry, which aims to develop geometric models of particle physics, are also a natural choice for a modified gravity model that incorporates the original Einstein–Hilbert action as well as correction terms involving conformal gravity and Gauss–Bonnet gravity and a series of higher-derivative terms related geometrically to the Seeley–DeWitt coefficients of the heat-kernel expansion of the Dirac Laplacian.

I will describe what one obtains from this spectral action functional for gravity for the ordinary homogeneous and isotropic Robertson–Walker metrics and where either homogeneity is maintained but not isotropy (such as Bianchi IX metrics [9, 10, 19]) or where isotropy is maintained but not homogeneity. The latter case is related to the possible fractality in the large-scale structure of space–time in the form of the so-called packed Swiss cheese cosmology models, which are based on Apollonian packings of homogeneous and isotropic space–times [2, 11, 18].

I will show how the use of the spectral action as a model of gravity also provides possible insights into the problem of cosmic topology (another question that has been quite popular in cosmology) through an effect of the topology of the three-dimensional spatial sections of space–time on the shape of the potential in slow-roll models of cosmic inflation.

Finally, I am going to discuss briefly how the terms in the asymptotic expansion of the spectral action functional — for sufficiently symmetric metrics like Robertson–Walker and Bianchi IX space–times — have interesting arithmetic structures involving periods of Tate motives and modular forms.

The topics I discuss in this article are the results of my collaboration with cosmologist Elena Pierpaoli, with my postdoc Farzad Fathizadeh, my graduate students Branimir Cacic and Kevin Teh, and Caltech undergraduate students Adam Ball, Wentao Fan and Yeorgia Kafkoulis. A general overview of this work and of other related results is available in my new book *Noncommutative Cosmology* [16]. The cosmological and astrophysical implications of the spectral action model of gravity have also been studied extensively by the physicist Mairi Sakellariadou and her collaborators.

Spectral gravity

The spectral action was introduced by Chamseddine and Connes [4] as a natural action functional associated with non-commutative spaces within the formalism of non-commutative Riemannian spin geometry based on the notion of a 'spectral triple.' The main idea is that the properties of an ordinary compact Riemannian spin manifold can be encoded axiomatically in the properties of its algebra $\mathcal{A}$ of smooth functions together with the Dirac operator D acting on the Hilbert space $\mathcal{H}$ of square integrable spinors. It is highly nontrivial [7] to show that a compact Riemannian spin manifold can be reconstructed from a spectral triple $(\mathcal{A}, \mathcal{H}, D)$ with an abelian algebra $\mathcal{A}$ satisfying an appropriate list of axioms detailing the properties of the Dirac operator and its interaction with the algebra. Extensions of this spectral-triple formalism from the commutative to the almost-commutative case (algebras that are locally products of finite-dimensional matrix algebras and commutative algebras of functions) have been successfully applied to the construction of geometric models of particle physics. See, for instance, the broad overviews presented in the book by van Suijlekom [20] and in Chapter 1 of [8].

It is important to point out here that the formalism of spectral triples, and the associated spectral action functional we will be reviewing shortly, apply to (the non-commutative geometry generalisation of) compact Riemannian geometries. Thus, from the point of view of gravity and cosmology, these would primarily provide models of Euclidean gravity (both in the classical context and in settings such as a Hartle–Hawking type version of Euclidean quantum gravity and quantum cosmology). However, despite the fact that the formulation of the spectral action itself is intrinsically Euclidean, the asymptotic expansion of the spectral action provides a sequence of terms given by local expressions in the curvature tensors, which can be made sense of also in cases where one can Wick rotate the geometry to Lorentzian. Thus, from the point of view of gravitational and cosmological models, one can view the Euclidean formulation as a necessary computational device, such that the resulting modified gravity terms can be extended to a Lorentzian setting.

Spectral action

The spectral action functional of a spectral triple $(\mathcal{A}, \mathcal{H}, D)$ is defined simply as a regularised trace of the Dirac operator D of the form

$$\mathcal{S}(D) = \mathrm{Tr}(f(D/\Lambda)), \quad (1)$$

where $\Lambda > 0$ is an energy scale, so that D/Λ is a dimensionless quantity, and f is an even test function on the real line that gives a smooth approximation to a cut-off function. The idea here is that, by cutting off the spectrum of the Dirac operator at a certain scale, one makes the trace finite, and in the limit when the scale $\Lambda \to \infty$, the contribution of the higher-energy modes of the Dirac operator is taken into consideration, thus probing the geometry at increasingly smaller scales.

Heat-kernel expansion

Let D^2 be the Dirac Laplacian of the spectral triple. If the heat kernel has an expansion at $\tau \to 0+$ of the form

$$\mathrm{Tr}(e^{-\tau D^2}) \sim \sum_\alpha c_\alpha \tau^\alpha, \quad (2)$$

where all the $\alpha > 0$ are integers, then this determines an asymptotic expansion for the spectral action functional, obtained as follows. For a test function of the form $f(x) = \int_0^\infty e^{-\tau x^2} d\mu(x)$ for some measure μ, the spectral action has an asymptotic expansion for $\Lambda \to \infty$ of the form

$$\mathcal{S}(D) \sim \sum_{\alpha<0} f_\alpha c_\alpha \Lambda^{-\alpha} + c_0 f(0) + \sum_{\alpha>0} f_\alpha c_\alpha \Lambda^{-\alpha}, \quad (3)$$

where

$$f_\alpha = \begin{cases} \int_0^\infty f(v) v^{-\alpha-1} dv, & \alpha < 0, \\ (-1)^\alpha f^{(\alpha)}(0), & \alpha > 0, \alpha \in \mathbb{N}. \end{cases}$$

This result implies that, to understand the large Λ behaviour of the spectral action model of gravity, one needs to understand the behaviour of the heat kernel of the Dirac Laplacian on the given geometry. The Mellin transform relation between the heat kernel and the zeta function of the Dirac operator also makes it possible to express the coefficients c_α in terms of poles and residues of the Dirac zeta function.

Modified gravity leading terms

For a four-dimensional smooth oriented compact spin Riemannian manifold, the leading terms

$$\mathcal{S}(D) \sim \frac{48 f_4 \Lambda^4}{\pi^2} \int \sqrt{g}\, d^4x + \frac{96 f_2 \Lambda^2}{24\pi^2} \int R \sqrt{g}\, d^4x + \frac{f_0}{10\pi^2} \int \left(\frac{11}{6} G(g) - 3 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \right) \sqrt{g}\, d^4x$$

in the asymptotic expansion of the spectral action functional contain, respectively, a cosmological term (volume form) with an effective cosmological constant, an Einstein–Hilbert action term with an effective gravitational constant, and modified gravity terms consisting of the Weyl curvature $C^{\mu\nu\rho\sigma}$ and a Gauss–Bonnet term $G(g)$, which is non-dynamical in dimension four (it integrates to the Euler characteristic of the underlying manifold). A suitable perturbation of the Dirac operator also introduces a conformal coupling of a scalar field to gravity as in the usual conformal gravity action functional.

Cosmic topology

A topic of interest to theoretical cosmology is the question of cosmic topology. Under the assumptions of a homogeneous and isotropic universe, the three-dimensional spatial sections of space–time would be either spherical space forms (S^3 or quotients by finite groups of isometries) in the positively curved case or Bieberbach manifolds (three-tori or quotients by finite groups of isometries) in the flat case. The negatively curved case of hyperbolic three-manifolds, while it is mathematically the richest class of three-dimensional geometries, is ruled out by current cosmological data, which favour the flat or slightly positively curved cases. General relativity by itself is sensitive to the curvature geometry but cannot distinguish between spaces that are locally isometric but differ in global topological properties. Thus, the question is whether there is detectable cosmological information that can distinguish between the different candidate cosmic topologies. Various methods have been tried, like statistically matching areas of the CMB map in an attempt to detect periodicities and attempts to justify in topological terms certain anomalies in the momenta of the expansion of the CMB signal in spherical harmonics. So far these methods have not led to conclusive evidence in favour of a particular topology, although some possibilities, like the Poincaré homology sphere, remain among the preferred candidates. For a general non-technical overview of the cosmic topology problem, see the book [14].

Under the hypothesis that the spectral action is a valid action functional for gravity that extends the Einstein–Hilbert action by modified gravity corrections, one can consider whether this model of gravity can capture some global information about the cosmic topology. The key idea here lies in that a perturbation $D^2 + \phi^2$ of the Dirac Laplacian by a scalar field ϕ introduces into the spectral action a potential $V(\phi)$ for the scalar field ϕ. This potential has a typical shape, with an asymptotic flat plateau at large scales and a quartic potential at small scales. This potential is used in cosmological models of inflation in the early universe, namely the slow-roll inflation models. According to these models, the field ϕ is responsible for the inflationary phase, which runs while the field is in the unstable equilibrium on the plateau and stops when it rolls down to its true vacuum. In these slow-roll models of cosmic inflation, the shape of the potential $V(\phi)$ determines certain slow-roll parameters:

$$\epsilon(\phi) = \frac{m_{\rm Pl}^2}{16\pi} \left(\frac{V'(\phi)}{V(\phi)} \right)^2,$$

$$\eta(\phi) = \frac{m_{\rm Pl}^2}{8\pi} \frac{V''(\phi)}{V(\phi)},$$

$$\xi(\phi) = \frac{m_{\rm Pl}^4}{64\pi^2} \frac{V'(\phi) V'''(\phi)}{V^2(\phi)},$$

where $m_{\rm Pl}$ is the Planck mass. These parameters combine to determine quantities like the spectral index n_s and the tensor-to-scalar ratio that are observable in the CMB data. The Fourier modes of the scalar and tensor (trace and traceless) perturbations h_{ij} of the metric satisfy power laws:

$$\mathcal{P}_s(k) \sim \mathcal{P}_s(k_0) \left(\frac{k}{k_0} \right)^{1 - n_s + (\alpha_s/2) \log(k/k_0)},$$

$$\mathcal{P}_t(k) \sim \mathcal{P}_t(k_0) \left(\frac{k}{k_0} \right)^{n_t + (\alpha_t/2) \log(k/k_0)},$$

where the exponents n_s, n_t, α_s and α_t depend on $V(\phi)$ through the slow-roll parameters. The amplitudes depend on $V(\phi)$ through

$$\mathcal{P}_s(k) \sim \frac{V^3}{(V')^2}, \quad \mathcal{P}_t(k) \sim V,$$

so that a global scaling $V \mapsto \lambda V$ of the potential induces the scalings $\mathcal{P}_s(k_0) \mapsto \lambda \mathcal{P}_s(k_0)$ and $\mathcal{P}_t(k_0) \mapsto \lambda \mathcal{P}_t(k_0)$ of the amplitudes.

For the spectral action functional of gravity, the slow-roll potential is determined by the spectral action through the perturbation of the Dirac Laplacian. An argument based either on a direct computation of the spectral action for the spherical space forms and Bieberbach manifolds using the Poisson summation formula [17] or on a heat-kernel analysis that includes the action of a finite group of isometries [3] shows that the shape of the potential $V(\phi)$ differs among the spherical space forms by an overall scaling factor that depends on the size of the isometry group, and similarly among the Bieberbach manifolds. On the other hand, the shape of the potential $V(\phi)$ (up to rescaling) differs more significantly between the flat and the positively curved cases in such a way that the corresponding slow-roll parameters differ between these two classes but remain constant within each class. The combination of these two factors shows that the slow-roll potential derived from the spectral action distinguishes between (most of) the candidate cosmic topologies.

Homogeneity and isotropy

Friedmann–Robertson–Walker

The Friedmann–Robertson–Walker space-times provide the standard form of a homogeneous and isotropic cosmological model. A scaling factor that depends on the cosmological time drives the expansion of the universe. In terms of the Euclidean signature, these metrics have the form $dt^2 + a(t)^2 d\sigma^2$, where $d\sigma^2$ is the standard round metric on the sphere S^3. Earlier work of Chamseddine and Connes [5] showed that the first few coefficients of the spectral action expansion on such a space-time can be computed explicitly as functions of the expansion factor $a(t)$ and its derivatives using the Feynman–Kac formula and Brownian bridge integrals. In [11], we showed that the same Brownian bridge technique, after a suitable reparameterisation, can be used to obtain closed formulae for all the terms and the complete expansion of the spectral action, which is expressed in terms of Bell polynomials and rational functions with rational coefficients in $a(t)$ and derivatives.

The Brownian bridge is a Gaussian stochastic process characterised by the covariance $\mathbb{E}(\alpha(v_1)\alpha(v_2)) = v_1(1-v_2)$ for $0 \le v_1 \le v_2 \le 1$. It is related to the heat kernel through the Feynman–Kac formula [5]:

$$e^{-sH_n}(t,t) = \frac{1}{2\sqrt{\pi s}} \int e^{-s\int_0^1 V_n(t+\sqrt{2s}\alpha(u))\,du}\, D[\alpha],$$

which can be applied to this setting after decomposing the Dirac Laplacian into a sum over eigenspaces of the Dirac operator on the three-dimensional sphere, $D^2 = \oplus_n H_n$ with $H_n = -d^2/dt^2 + V_n(t)$, where the potential $V_n(t)$ depends on the scaling factor $a(t)$. The terms of the heat-kernel expansion for the Robertson–Walker metric can then be written in the form of Brownian bridge integrals [11]:

$$\mathrm{Tr}(\exp(-\tau^2 D^2)) \sim \sum_{M=0}^{\infty} \tau^{2M-4} \int a_{2M}(t)\,dt,$$

where the $a_{2M}(t)$ are of the form

$$\int \left(\frac{1}{2} C_{2M}^{(-3/2,0)} + \frac{1}{4}\left(C_{2M-2}^{(-5/2,2)} - C_{2M-2}^{(-1/2,0)}\right)\right) D[\alpha].$$

Here the $C_{2M}^{(r,n)}$ terms, which provide the series for all the modified gravity corrections, are explicit expressions in the Bell polynomials evaluated at the terms u_i and v_j of the Taylor expansions

$$U = \tau^2 \sum_{n=0}^{\infty} \frac{u_n}{n!}\tau^n \quad \text{and} \quad V = \tau^2 \sum_{n=0}^{\infty} \frac{v_n}{n!}\tau^n$$

in the heat-kernel variable $\tau = s^{1/2}$ of the expressions (with $A(t) = 1/a(t)$)

$$U = s\int_0^1 A^2\left(t + \sqrt{2s}\alpha(v)\right)\,dv,$$

$$V = s\int_0^1 A'\left(t + \sqrt{2s}\alpha(v)\right)\,dv,$$

with $\alpha(v)$ the Brownian bridge variable. In this heat-kernel expansion, if the Robertson–Walker metric is scaled by an overall factor a, each coefficient $C_{2M}^{(r,n)}$ is scaled by a corresponding power a^{4-2M}. The Bell polynomials have a combinatorial structure that arises naturally in the recursive formulae for derivatives of composite functions expressing $\frac{d^n}{dt^n} f(g(t))$ as a sum, for $m = 1, \dots, n$, of terms $f^{(m)}(g(t))\, B_{n,m}(g'(t), g''(t), \dots, g^{(n-m+1)}(t))$ and in the associated Faà di Bruno Hopf algebra encoding these relations.

Packed Swiss cheese cosmology

Other new and interesting structures emerge when one drops either the homogeneity or the isotropy hypothesis of the Friedmann–Robertson–Walker models. In the isotropic non-homogeneous case of the packed Swiss cheese cosmology model, one can consider an Apollonian packing $\mathcal{P}$ of spaces, each of which is a properly scaled compactified Robertson–Walker space-time that is topologically isomorphic to a four-dimensional sphere [2, 11, 18]. The resulting space has a non-integer fractal dimension, which can be estimated in terms of the exponent of convergence of the series $\zeta_{\mathcal{L}}(s) = \sum_{n,k} a_{n,k}^s$, with $n \geq 1$ the iterative levels in the construction of the packing. k ranges over the number of spheres in the nth level of the packing, and $a_{n,k}$ is the sequence of radii of the spheres in the packing. The reader should consult [13] for the general properties of these Apollonian packings of higher-dimensional spheres.

The spectral action on such a packing $\mathcal{P}$ can be computed in terms of the spectral action on an individual Robertson–Walker space-time together with a Mellin transform argument. A spectral triple for an Apollonian packing of spheres can be constructed using the method of [6] such that the resulting Dirac operator is a direct sum of the Dirac operators on the individual spheres. Note that, although the packing is no longer a smooth manifold, the formalism of spectral triples is so designed that one can continue to make sense of a good notion of a Dirac operator on spaces that are not manifold-like, such as fractals and non-commutative spaces. To compute the heat-kernel expansion of the corresponding Dirac Laplacian, one can use the relation between asymptotic expansions and singular expansions via a Mellin transform. In general, given a function $f(\tau)$ with small-time asymptotics:

$$f(\tau) \sim \sum_N c_N \tau^N,$$

and a sequence $R = \{r_n\}$ of scaling factors $r_n \in \mathbb{R}_+^*$ for which the zeta function $\zeta_R(z) = \sum_n r_n^{-z}$ converges in the half-plane $\Re(z) > C$ for some $C > 0$, the series $g_R(\tau) = \sum_n f(r_n \tau)$ has small-time asymptotics:

$$g_R(\tau) \sim_{\tau \to 0^+} \sum_N c_N \zeta_R(-N) \tau^N + \sum_{\sigma \in \mathcal{S}(\zeta_R)} \mathcal{R}_{R,\sigma} \mathcal{M}(f)(\sigma) \tau^{-\sigma},$$

where $\mathcal{M}(f)$ is the Mellin transform and $\mathcal{S}(\zeta_R)$ is the set of poles of $\zeta_R(z)$ with residues

$$\mathcal{R}_{R,\sigma} := \mathrm{Res}_{z=\sigma} \zeta_R(z).$$

Applied to the sequence of radii of the Apollonian packing and to the asymptotics of the heat kernel of the Dirac Laplacian on a single Robertson–Walker space-time, this method provides the asymptotic expansion for the heat kernel (hence, for the spectral action) on the packed Swiss cheese cosmology $\mathcal{P}$.

There are two main new effects through which this gravity model detects the presence of fractality in the packed Swiss cheese cosmology, even though these space-times look locally Robertson–Walker. The first occurs in the coefficients $C_{2M}^{(r,n)}$ of the Robertson–Walker spectral action expansion, which is now multiplied by a zeta regularisation $\zeta_{\mathcal{L}}(4 - 2M)$ of the series of the powers $a_{n,k}^{4-2M}$ of the radii of the packing. These in particular affect the effective gravitational and cosmological constants of the model. The other effect involves new terms that are not in the asymptotic expansion of the Robertson–Walker case. These terms depend on the poles and residues of the zeta function $\zeta_{\mathcal{L}}(s)$ of the Apollonian packing. This zeta function, in general, has poles off the real line. Hence, the corresponding terms in the spectral action expansion contain log periodic terms and an overall order $\Lambda^{\Re\sigma}$, with $3 < \Re\sigma = \dim_H \mathcal{P} < 4$. Thus, in terms of scales, these fractality correction terms to the gravity action functional occur between the Einstein–Hilbert and the cosmological term.

Bianchi IX

When homogeneity is preserved but not isotropy, there is a different picture. An important class of (Euclidean signature) space-times with these properties are the Bianchi IX metrics, which have three different anisotropic scaling factors and are based on metrics on the three-sphere that are no longer both left and right $SU(2)$ invariant like the round metric but preserve only one of these symmetries. The general form of the Bianchi IX metrics is

$$g = F\left(d\mu^2 + \frac{\sigma_1^2}{w_1^2} + \frac{\sigma_2^2}{w_2^2} + \frac{\sigma_3^2}{w_3^3}\right), \tag{4}$$

with scaling factors $w_i = w_i(\mu)$ that are functions of the cosmological time μ and with a conformal factor $F \sim w_1 w_2 w_3$. The σ_i are a basis of $SU(2)$-invariant 1-forms on S^3 with $d\sigma_i = \sigma_j \wedge \sigma_k$ for all cyclic permutations of indices. The Euclidean Bianchi IX space-times play an important physical role as mini-superspace models. There are some especially interesting metrics in this family, namely the *gravitational instantons*, which are both Einstein and self-dual. It is known that the Bianchi IX gravitational instantons can be classified in terms of solutions to a Painlevé VI equation and can be parameterised in terms of theta functions [1, 19]. In particular, there is a two-parameter family $(w_i[p,q], F[p,q])$ of solutions that are gravitational instantons with a non-vanishing cosmological constant, expressed in terms of the theta characteristic $\vartheta[p,q](0,i\mu)$, and a one-parameter family of solutions with vanishing cosmological constant.

As in the Robertson–Walker case, for the Bianchi IX metrics (4) one can also show [9] that in the coefficients $\int a_{2n}(\mu)\,d\mu$ of the heat-kernel expansion, the terms $a_{2n}(\mu)$ are rational functions:

$$\frac{Q_{2n}\left(w_1, w_2, w_3, F, \ldots, w_1^{(2n)}, w_2^{(2n)}, w_3^{(2n)}, F^{(2n)}\right)}{F^{2n}(w_1 w_2 w_3)^{3n-1}},$$

where the Q_{2n} are polynomials with rational coefficients in the scaling factors w_i and their derivatives and the conformal factor F and its derivatives.

For the Bianchi IX gravitational instantons, moreover, the parameterisation of w_i and F in terms of theta characteristics determines a modularity property for the coefficients of the heat-kernel expansion (hence, of the spectral action) in terms of a complexified cosmological time $i\mu$ [9]:

$$a_{2n}[p,q](i\mu+1) = a_{2n}\left[p, q+p+\frac{1}{2}\right](i\mu),$$

$$a_{2n}[p,q]\left(\frac{i}{\mu}\right) = (i\mu)^2 a_{2n}[-q,p](i\mu),$$

and similarly for the vanishing cosmological constant case. One can, therefore, think of the coefficients of the spectral action expansion in this case as vector-valued modular forms.

Arithmetic phenomena

The occurrence of modular forms in the spectral action expansion for the Bianchi IX gravitational instantons is an instance of a broader range of arithmetic phenomena involving the coefficients of the heat kernel on, in particular, symmetric space–time geometries such as the Robertson–Walker and the Bianchi IX metrics discussed above.

It is well known that many asymptotic expansions in physics, especially those that arise in perturbative quantum field theory through the expansion of Feynman diagrams, have individual terms (or larger combinations of terms) that can be identified with periods of certain algebraic varieties. See [15] for an overview.

The periods of algebraic varieties are numbers (usually transcendental) obtained from the integration of an algebraic differential form on a locus on the variety cut out by algebraic equations. Numbers such as multiple zeta values are known to occur as periods. In general, what numbers can occur as periods is constrained by the geometry of the variety through its *motive*. Motives were introduced by Grothendieck in the 1960s as a universal cohomology theory for algebraic varieties. The development of a good theory of motives involves several deep conjectures about algebraic cycles and remains a very active field of research today. Pure motives cover smooth projective varieties whereas more general varieties fall within the theory of mixed motives developed by Voevodsky and others in more recent years. Dealing directly with objects in the category of motives is often difficult, but information about a motive can be gained from a simpler associated invariant, the Grothendieck class, which can be seen as a universal Euler characteristic for algebraic varieties. The Grothendieck ring of varieties is generated by isomorphism classes $[X]$ with the inclusion–exclusion relations $[X] = [X \smallsetminus Y] + [Y]$ for embedded subvarieties $Y \hookrightarrow X$ and with the product $[X][Y] = [X \times Y]$.

A particularly interesting subcategory of mixed motives consists of the mixed Tate motives. Heuristically, one can think of these as motives built out of pieces that "look like projective spaces" (the pure Tate motives) through non-trivial extensions in the triangulated category of mixed motives. The Grothendieck classes of the Tate motives are polynomial functions with integer coefficients in the Lefschetz motives $\mathbb{L} = [\mathbb{A}^1]$, the class of the affine line. For the periods and motives that arise in scalar perturbative quantum field theories, the motives and periods associated with the Feynman diagrams remain mixed Tate for small graphs but eventually involve non-mixed-Tate cases as well, if one goes sufficiently deep into the expansion.

Coming back to our theory of gravity based on the spectral action functional, it is shown in [10, 12] that, in a suitable set of coordinates, the integrals describing the coefficients in the spectral action expansion (and in the heat-kernel asymptotic expansion for the Dirac Laplacian) on both the

Robertson–Walker and the Bianchi IX metrics are all periods of mixed Tate motives. More precisely, there is a natural change of coordinates that replaces combinations of trigonometric functions appearing in the parameterisation of the metric and the Dirac operator with polynomials and rational functions with rational coefficients. Moreover, the scaling factor $a(t)$ and its derivatives $a^{(i)}(t)$, $i = 1,\ldots,2n$, in the Robertson–Walker case are regarded in this period computation as independent affine variables α, α_i, $i = 1,\ldots,2n$, and similarly for the parameters $w_i(t)$ and their derivatives in the Bianchi IX case. The integrand for the term a_{2n} of the heat-kernel expansion is then an algebraic differential form $\Omega^{\alpha}_{\alpha_1,\ldots,\alpha_{2n}}(u_0, u_1, \ldots, u_{2n+2})$, which is defined on the complement in the affine space $\mathbb{A}^{2n+3}$ of the union of two hyperplanes H_0 and H_1 and a cone $\widehat{CZ}_{\alpha,2n}$ over the quadric defined by the form

$$Q_{\alpha,2n} = u_1^2 + \frac{1}{\alpha^2}(u_2^2 + u_3^2 + u_4^2) + u_5^2 + \cdots + u_{2n+2}^2.$$

Thus, the underlying algebraic varieties for this period interpretation are the complements

$$\mathbb{A}^{2n+3} \smallsetminus (\widehat{CZ}_{\alpha,2n} \cup H_0 \cup H_1), \tag{5}$$

which are viewed as objects in the category of mixed motives. The domain of integration is a $\mathbb{Q}$-semi-algebraic set A_{2n+2} defined as the real locus in $\mathbb{A}^{2n+3}(\mathbb{R})$ cut out by equations $u_1^2 + u_2^2 + u_0 u_3^2 + (1 - u_0)u_4^2 + \sum_{i=5}^{2n+2} u_i^2 = 1$ with $0 < u_i < 1$ for $i = 0,1,2,5,6,\ldots,2n+2$. The period integral is convergent. Hence, unlike the quantum field theory case, no regularisation or renormalisation issues arise. The nature of the motive of the variety (5) can be understood either using distinguished triangles in the Voevodsky category of mixed motives and the property that if two of the three terms in such a triangle are mixed Tate then the third one should also be (Tate motives are a triangulated subcategory) or else by explicitly computing the class in the Grothendieck ring of the variety (5). The Grothendieck class can be computed explicitly. In the Robertson–Walker case, the class associated with the $2n$th term in the asymptotic expansion in the spectral action is given by

$$[\mathbb{A}^{2n+3} \smallsetminus (\widehat{CZ}_{\alpha,2n} \cup H_0 \cup H_1)] = \mathbb{L}^{2n+3} - 3\mathbb{L}^{2n+2} + 2\mathbb{L}^{2n+1} - \mathbb{L}^{n+2} + 3\mathbb{L}^{n+1} - 2\mathbb{L}^{n}.$$

A similar structure can be seen for the Bianchi IX metrics. In this case, the a_{2n} term of the heat-kernel expansion is written as a period integral of an algebraic differential form defined on a variety:

$$\mathbb{A}^{2n+4} \smallsetminus (H_+ \cup H_- \cup \widehat{C^2Z}_{W,2n}),$$

which is the complement in an affine space of a union of two hyperplanes $H_\pm$ and a hypersurface $\widehat{C^2Z}_{W,2n}$, which is obtained by taking the affine cone of the projective cone $C^2Z_{W,2n}$ in $\mathbb{P}^{2n+3}$ of the quadric in $\mathbb{P}^{2n+1}$ determined by the quadratic form:

$$Q_{W,2n}(\zeta_1,\ldots,\zeta_{2n+2}) = \sum_{i=1}^{4} W_i^2\zeta_i^2 + \sum_{i=5}^{2n+2}\zeta_i^2,$$

where the W_i are affine variables representing the anisotropic scaling factors. The locus of integration is a semi-algebraic set defined in $\mathbb{A}^{2n+4}(\mathbb{R})$ by the relations $\sum_{i=1}^{2n+2}\zeta_i^2 = 1$ and $0 < \mu_1, \mu_2 < 1$. The Grothendieck class in this case is given explicitly by

$$[\mathbb{A}^{2n+4} \smallsetminus (\widehat{C^2Z} \cup H_+ \cup H_-)] = \mathbb{L}^{2n+4} - 3\mathbb{L}^{2n+3} + 2\mathbb{L}^{2n+2} - \mathbb{L}^{n+3} + 3\mathbb{L}^{n+2} - 2\mathbb{L}^{n+1}.$$

An argument based on distinguished triangles in the triangulated category of mixed motives can also be used to show that these motives are all mixed Tate.

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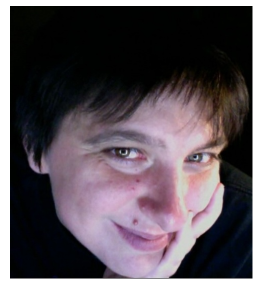

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A Leibniz/NSA Comparison

MIKHAIL G. KATZ, KARL KUHLEMANN, AND DAVID SHERRY

We present some similarities between Leibnizian and Robinsonian calculi and address some objections raised by historians. The comparison with non-standard analysis (NSA) facilitates our appreciation of some Leibnizian procedures that may otherwise seem obscure. We argue that Leibniz used genuine infinitesimals and infinite quantities that are not merely stenography for Archimedean exhaustion and that Leibniz's procedures, therefore, find better proxies in NSA than in modern Weierstrassian mathematics.

Leibniz and NSA

The Leibnizian infinitesimal calculus represents a different approach to the calculus from the one generally adopted today. A seminal study of Leibnizian methodology was published by Bos [6]. A number of distinct interpretations of the Leibnizian calculus have been pursued by Leibniz scholars over the past few decades.

Table 1. Comparison of Leibniz and NSA

Leibniz	NSA
Relation of infinite proximity	Standard part (shadow)
Law of continuity	Transfer principle
Assignable vs inassignable number	Standard vs non-standard number
Infinitum terminatum	Line segment of unlimited length
Infinitesimal violates Euclid V.4	Infinitesimal violates the Archimedean property
Constant differentials	Uniform partition into infinitesimal subsegments

The article [12], which analysed a pair of rival interpretations of the Leibnizian calculus, mentioned neither Robinson nor NSA, although the article made a passing mention of non-standard models of arithmetic that go back to Skolem [22]. The present text seeks to make the Leibniz/NSA comparison more explicit. We mainly follow the interpretation of Leibniz given by Bos [6].

In "Six similarities," we summarise some relevant procedures of the Leibnizian calculus and signal some similarities with the Robinsonian calculus. The similarities are summarised in Table 1. In "Possible objections," we deal with some objections by historians.

Six similarities

We present six similarities between the Leibnizian and Robinsonian calculi, as summarised in Table 1.

What kind of equality?

Leibniz mentioned that he worked, not with the relation of *exact* equality between quantities, but rather equality up to negligible infinitesimal terms, as when he wrote:

> I think that not only those things are equal whose difference is absolutely zero, but also those whose difference is incomparably small. And although this [difference] need not absolutely be called Nothing, neither is it a quantity comparable to those whose difference it is. [16, p. 322]

This can be described as a relation of *infinite proximity*. In NSA, a related procedure involves extracting the *standard part* of a limited quantity (a number is *limited* if it is smaller in absolute value than some standard number).

Law of continuity

One of the formulations of Leibniz's *law of continuity* is "the rules of the finite are found to succeed in the infinite and vice versa" ("Il se trouve que les regles du fini reussissent dans l'infini, ... et que vice versa les regles de l'infini reussissent dans le fini") [18, pp. 93–94]. (Compare [21, p. 266].) In reference to Leibniz's law, Robinson asserted the following:

> This is remarkably close to our transfer of statements from $\mathbb{R}$ to ${}^*\mathbb{R}$ and in the opposite direction. [21, p. 266]

The transfer principle is a theorem asserting that a formula that holds for all standard inputs would, in fact, hold for all inputs. Thus, if we had a proof that a relation such as $\cos^2 x + \sin^2 x = 1$ holds for all standard x, it would automatically hold for all x by the transfer principle.

Assignable or inassignable?

Leibniz mentioned frequently a distinction between assignable and inassignable numbers, as when he writes:

> Here dx means the element, that is, the (instantaneous) increment or decrement, of the (continually) increasing quantity x. It is also called difference, namely the difference between two proximate x's which differ by an element (or by an *inassignable*), the one originating from the other, as the other increases or decreases (momentaneously). [Leibniz as translated by Bos, 6, p. 18, emphasis added]

In NSA, a parallel distinction exists between standard and non-standard numbers. The distinction is used routinely in the procedures of modern infinitesimal analysis. Thus, an infinitesimal is defined as a number smaller in absolute value than every standard positive real number, and the continuity of a function f at a standard point x is defined by the condition that an infinitesimal change α of the independent variable always produces an infinitesimal change $f(x+\alpha) - f(x)$ of the function.

What is bounded infinity*?*

Starting in his 1676 *De Quadratura Arithmetica* [19], Leibniz used a type of line segment he referred to as *infinitum terminatum*, which literally means "bounded infinity." This is greater than any assignable line segment. What could the expression *bounded infinity* possibly mean? In his chapter on the history of mathematics [21, Chapter 10], Robinson makes no mention of Leibniz's use of such an expression. To a modern reader trained in the Weierstrassian paradigm, the expression *bounded infinity* may seem like a contradiction in terms. The pioneering work of Cantor, Weierstrass and others in creating rigorous foundations for analysis has conditioned successive generations of mathematicians to hold that there are just two types of infinite number in mathematics: *ordinals* and *cardinals*. (To Leibniz, such concepts of modern set theory would arguably have been meaningless, since he rejected the notion of an *infinite whole* as contradictory. To Leibniz, *bounded infinity* was the consistent counterpart of *unbounded infinity*, such as the unbounded real line in modern terminology. For a further analysis, see [2].)

It is helpful to go beyond that dichotomy and allow also for the possibility of what could be called *ringinals*, namely, elements in a number system that are greater than all naive integers (such as a non-standard integer in the semiring ${}^*\mathbb{N}$ extending $\mathbb{N}$). Such numbers are referred to as *unlimited*.

Leibniz's notion of *infinitum terminatum* is analogous to segments of unlimited length (on a non-standard line), which are greater than segments of any standard length. The notion played a crucial role in the Leibnizian calculus, by enabling non-Archimedean phenomena (see "Leibniz on violation of Euclid Book V, Definition 4"). Thus, an infinitesimal is

> [an] infinitely small fraction, or one whose denominator is an infinite number ("fraction infiniment petite, ou dont le denominateur soit un nombre infini") [18, p. 93]

The rules for operating with infinitesimal differentials and their applications to developing the basic procedures of the calculus, including the Leibniz rule for the product, first appeared in Leibniz's 1684 article [15].

Leibniz on violation of Euclid Book V, Definition 4

In a 1695 article [16] containing a rebuttal of a criticism by Nieuwentijt, Leibniz made it clear that his "incomparable" quantities, when compared to 1, violate the comparability notion formulated in Euclid

Book V, Definition 5 (appearing as Definition 4 in modern editions). Euclid's definition reads as follows:

> Any magnitude may be multiplied as many times as to exceed any given homogeneous magnitude. [8, p. 626]

Leibniz made similar comments in a famous 1695 letter to l'Hôpital [17, p. 288]. Similarly, NSA infinitesimals violate the Archimedean property.

Constant differentials

Leibniz historians often mention his use of terms such as "progression of variables" and "constant differentials." Thus, Bos writes:

> Leibniz explains that if dx is taken constant, one may treat the quadrature as $\int y$ ("sum of all y"), as is done in the theory of indivisibles, but if one wishes to consider different progression of the variables, the quadrature has to be evaluated as $\int y\,dx$. [6, p. 27]

Such comments can be formalised in modern infinitesimal analysis in terms of a uniform partition of the segment into infinitesimal subsegments, when the integral can be computed via the Riemann sum corresponding to the uniform partition. (Of course, the distinction between uniform and non-uniform partitions of the interval of integration can equally well be formulated in traditional non-infinitesimal analysis; however, the latter lacks the language to express partitions into *infinitesimal* subsegments, which would necessarily have to be paraphrased in terms of limits of sequences of Archimedean partitions.)

Then 'non-constant differentials' would correspond to partitions that are not necessarily uniform. The idea of 'progression of the variables' may seem odd to a reader accustomed to thinking of a variable as ranging through a continuous range such a subinterval of $\mathbb{R}$, but it is natural from the viewpoint of infinitesimal partitions exploited in NSA.

Possible objections

Among historians of mathematics, critical attitudes are not uncommon towards attempts to mention Leibniz and NSA in the same article. In the context of a discussion of the Leibnizian calculus, a noted historian expressed the following reservation: "The view that NSA is pervasive in the history of math is a legitimate (though wrong, to my view) thesis that anyone has the right to defend" (Marco Panza, private communication, 17 March 2023). There are at least three possible ways of interpreting the claim that would be the intended target of such a reservation:

(1) Leibniz used ultrafilters.

(2) Leibniz's procedures find better proxies in NSA than in modern Weierstrassian mathematics.

(3) Leibniz's infinitesimals were non-Archimedean and not merely stenography for Archimedean exhaustion.

Clearly, claim (1) would be an anachronism, whether it is made with regard to ultrafilters and ultrapowers or other modern set-theoretic constructions of non-standard number fields.

With regard to claim (2), it would perhaps be reasonable to maintain that Leibniz was neither Weierstrassian nor Robinsonian, while both can be helpful for understanding aspects of his work. Nevertheless, one can analyse the *procedures* of the Leibnizian calculus to gauge which modern theory provides better proxies for the kind of inferential moves used by Leibniz. Thus, the importance of the notion of the *infinitum terminatum* in the Leibnizian calculus and geometry (see "What is *bounded infinity*?") was emphasised in the early articles by Leibniz historian Knobloch [13, 14]. As mentioned in "Leibniz and NSA," there is a ready analogue in NSA, given by segments of unlimited length such as a non-standard integer. Finding proxies in a Weierstrassian framework would be more difficult and necessarily involve an infinite process rather than a single segment.

With regard to claim (3), certainly Leibniz was no mere stenographer. More to the point, one can ask whether, whenever Leibniz used the term *infinitesimal*, he just meant the reader to fill in the details of the corresponding Archimedean exhaustion argument, as in an Ishiguroan interpretation [11, Chapter 5], which reads the passages containing the term *infinitesimal* as shorthand for Archimedean exhaustion; this particular debate was already analysed in *Depictions* [12]. Leibniz scholarship today is an area of lively debate; see, e.g., Archibald et al. [1], Bair et al. [4], and Ugaglia and Katz [23].

We comment briefly on possible interpretations of the work of other historical mathematicians in NSA terms. While Cauchy often used genuine infinitesimals, as detailed in [3] and [4], he had less interest in philosophy than Leibniz. Cauchy produced mathematics for the most part without looking for underlying philosophical principles (with the possible exception of his Torino lectures [7]). While there are a few mentions of assignable quantities in Cauchy, one does not find any mention of inassignables and certainly no occurrences of *infinita terminata*.

Leibniz viewed principles such as the law of continuity as philosophical insights. Today we view its counterpart, the transfer principle, as a mathematical insight. Cauchy was not interested in such general principles as much as Leibniz, and therefore, from a modern viewpoint, did not get as far as Leibniz foundationally speaking, even though he was, of course, much ahead of Leibniz mathematically speaking.

As far as Fermat is concerned, he was extremely cautious, and we have little indication of what he thought about his increments E. Indeed, there were good reasons for his caution [5]. We have argued that adequality is best understood as some kind of relation of infinite proximity [see also 5].

Conclusion

The viability of interpreting the Leibnizian calculus in terms of NSA depends crucially on the distinction between procedures and foundations. The analogies we developed here concern the realm of procedures (such as those listed in Table 1). As far as foundations are concerned, it may be pertinent to mention the existence of (at least) two distinct approaches to NSA. Robinson's approach, as developed in the 1960s, involved extending the field of real numbers $\mathbb{R}$ to the field of hyperreals ${}^*\mathbb{R}$ [21].

In the 1970s, Hrbacek [10] and Nelson [20] independently developed axiomatic approaches to NSA. In such axiomatic approaches, infinitesimals are found, not in an extension ${}^*\mathbb{R}$, but rather in $\mathbb{R}$ itself. What enables this is the introduction of a one-place predicate 'standard' postulating the existence of two types of real numbers (and other entities): standard and non-standard.

In axiomatic approaches, an infinitesimal is a number smaller in absolute value than every positive standard real number. Hrbacek and Nelson showed the conservativity of their theories over the traditional Zermelo–Fraenkel set theory with the axiom of choice, meaning that the introduction of the new predicate did not involve any new foundational commitments and did not, in principle, prove any new theorems. If we were to go beyond our analysis of the Leibnizian procedures and ask which foundational approach to NSA would be closer to the spirit of the Leibnizian calculus, we would probably have to answer that the axiomatic approach is closer, since the existence of inassignable infinitesimals is postulated by Leibniz, rather than resulting from any kind of construction (as in Robinson's original approach).

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Mikhail G. Katz

Mikhail Katz (BA Harvard 1980; PhD Columbia 1984) is Professor of Mathematics at Bar-Ilan University, Israel. One of his recent articles is "Effective infinitesimals in ℝ" (2023, joint with Karel Hrbacek) in *Real Analysis Exchange*.

Karl Kuhlemann

Karl Kuhlemann studied mathematics and computer science at the University of Münster (diploma degree in 1991), worked at IBM and Finanz Informatik, where he is still employed today. He received his doctorate in 2022 from the Leibniz University Hannover with a thesis on NSA. An English version of his book on NSA is scheduled to be published by de Gruyter at the end of 2024.

David Sherry

David Sherry is Professor of Philosophy Emeritus at Northern Arizona University, among the tall pines of the Colorado Plateau. He received his bachelor's degree from the University of Montana and his PhD from Claremont Graduate School. His current research interests include philosophical issues arising from biological classification and explanation in natural history. He spends inordinate amounts of time trying to play Bach on his mandolin.

A Postcard from Seville

MARIANNE FREIBERGER

The *9th European Congress of Mathematics (ECM) 2024* took place in Seville in July. Nearly 1,400 participants from all over the world gathered in this beautiful Spanish city to learn about the latest advances in pure and applied mathematics. The ECM is organised by the European Mathematical Society (EMS) and takes place every four years. A wide variety of topics were covered, including artificial intelligence, education, big data and open science. In the words of the chair of the Organising Committee, Juan González-Meneses, the topics showed "that the mathematics being developed today will change the life of tomorrow."

Flamenco and fame

The Congress kicked off on 15 July with an opening ceremony at Seville's Maestranza Theatre, which is more used to hosting music, dance and opera. The ceremony did feature a stunning performance of modern flamenco music and dance by the group Dorantes, but most of the limelight was on mathematicians — those who had organised the conference and, excitingly, the recipients of 14 prizes that were awarded at the event.

Ten of these, known as the EMS Prizes, are awarded at each ECM to early career researchers not older than 35 at the time of nomination. The EMS Prizes are highly prestigious, not least because they often indicate greater things to come. As Pavel Exner, chairman of the Awards Committee said: "The EMS Prizes have become very well known, and many of the winners have been also awarded with Fields Medals." He added that: "This year there have been many more nominees than in previous years, and this has been a great joy and also a frustration — we've seen [such] great mathematics from young people, but we had to make hard choices."

Graphs and pyjamas

Three of this year's prizes went to British mathematicians. Richard Montgomery, Associate Professor at the University of Warwick, received the prize for "his solution of the Ringel tree packing conjecture, development of distributive absorption techniques with applications to graph embedding problems, and resolution of several classical conjectures of Erdös and others on cycle lengths in sparse graphs using the novel machinery of sublinear expanders."

Ringel's conjecture is a particularly neat result. Posed in 1963 by Gerhard Ringel, the conjecture says that the complete graph on $2n+1$ vertices can be decomposed into edge-disjoint copies of a tree with n edges. And not only that. *Any* tree with n edges, so it was conjectured, can cover the complete graph in this way. Montgomery, together with Alexey Pokrovskiy and Benny Sudakov, proved the result for large n.

Another British mathematician who received an EMS Prize is Thomas Hutchcroft, Full Professor at the California Institute of Technology, "for his revolutionary contributions to probability theory and geometric group theory, in particular to percolation theory on general graphs, using tools from geometry, operator theory, group theory and functional analysis."

Percolation theory investigates the behaviour of graphs as edges are added or taken away. You might start with a grid of points and link neighbouring points with probability p. If $p = 0$, no points are connected, and if $p = 1$, all neighbouring points are connected. The interesting feature is that, for certain types of infinite graph, the system undergoes a geometric phase transition at a critical probability p_c. Loosely speaking, for $p > p_c$ the graph contains a giant connected component, but as p falls to below p_c, the graph fragments into many small, connected clusters.

Percolation theory has interesting applications. For example, during the COVID-19 pandemic, disease modellers used a percolation model[1] to see what proportion of social contacts should better not happen to stop the infection from sweeping through the entire population. Here the vertices in the graph represented households and edges social contacts

[1] plus.maths.org/content/careful-your-christmas-baubbles

between them. The idea was to find the critical value p_c at which the network breaks up into fragments, so that any outbreaks would remain confined to those fragments.

Applications aside, percolation theory offers rich and deep mathematics. One interesting feature, called *universality*, pertains to the fact that, near the critical probability, many features of the models depend only on the large-scale geometry of the graphs. While universality features in Hutchcroft's work, his EMS Prize citation also mentions a result that goes in the opposite direction. The *Schramm locality conjecture*, posed in 2008 by Oded Schramm, suggests that the critical probability p_c may be determined entirely by the small-scale geometry of the graph. Hutchcroft, together with Philip Easo, proved the conjecture for certain types of percolation models.

Another Brit among the EMS prize winners was Frederick Manners, Professor at the University of California, San Diego. He received the prize "for his remarkable contributions to additive combinatorics and related areas, in particular to the foundations of higher-order Fourier analysis, as well as for miscellaneous other results such as the solution of the pyjama problem." The latter imagines vertical stripes in the plane at integer x-coordinates and asks whether finitely many rotations of the "pyjama stripe pattern" around the origin can cover the plane. Manners proved the answer is yes, no matter how thin the original stripes.

Funding for five of the ten EMS Prizes is provided by the Foundation Compositio Mathematica. Any surplus income arising from the publication of *Compositio Mathematica* is invested back into mathematics through its charitable activities.

Apart from the EMS Prizes, the ECM also saw the award of four other important prizes. The Felix Klein Prize for an "outstanding solution of a concrete and difficult industrial problem" went to Fabien Casenave of Safran Tech, Digital Sciences & Technologies. The Otto Neugebauer Prize for "highly original and influential work in the field of history of mathematics" went to Reinhard Siegmund-Schultze, Professor Emeritus at the University of Agder, Norway. The Paul Lévy Prize in probability theory was awarded to Jeremy Quastel, Full Professor at the University of Toronto. And the EMS/ECMI Lanczos Prize for the development of outstanding mathematical software was awarded to the MUMPS (MUltifrontal Massively Parallel Sparse direct Solver) software library, developed by Patrick Amestoy, Jean-Yves L'Excellent and Chiara Puglisi. For a full list of this year's prize winners, see the EMS website: tinyurl.com/bdcm9rps.

Zero knowledge and backstreet groups

After the prizes were announced at the opening ceremony, the delegates dived into a week of mathematical adventure, featuring 12 plenary and special lectures, around 30 invited lectures, 64 mini-symposia and 270 contributed communications. There were also round tables, outreach talks, activities for young researchers and a cultural programme, including four exhibitions and a visit to Seville's beautiful Alcázar Palace. One hundred mathematics students from several Spanish universities were at hand as volunteers. For lunch, we were treated to delicious tapas.

What was noticeable throughout the week was the high quality of the talks, which were accessible, informative and entertaining for a general mathematical audience. One personal highlight was the Abel Lecture by Avi Wigderson (IAS), who explored what can happen when you allow errors into a proof system. Rather than requiring a *verifier* to accept only correct proofs presented by a *prover*, you allow a tiny chance of the verifier getting it wrong.

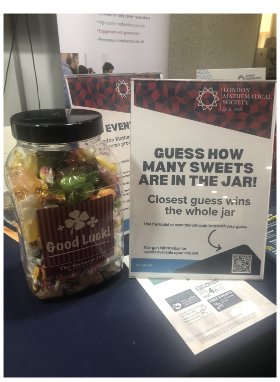
Interesting things happen as a result of this relaxation. For example, in such a system, suitably formalised and also equipped with the feature that the verifier can interact with the prover, all classical proofs can be transformed into *zero-knowledge proofs*. These are arguments that convince the verifier but teach them nothing else. As an analogy, imagine me convincing you that I know the location of a buried treasure without disclosing its location. As Wigderson told me in an interview, in such a proof system "Andrew Wiles, if he wanted to, could convince us that he [had proved Fermat's last theorem] without telling the world how."

Another highlight was the plenary lecture by Martin Bridson (University of Oxford) entitled *Chasing Finite Shadows of Infinite Groups Through Geometry*. Bridson pointed out that groups are as ubiquitous in mathematics as numbers are, so they do not just belong to algebraists. To really understand a group, he said, let it act on a space. Over the course of a very entertaining hour, Bridson told us how we might find out whether an infinite group we have found on the streets of Seville is isomorphic to our favourite group.

Britain in Europe

Figure 1. Jens Marklof introduces Heather Harrington, speaker at the LMS lecture

The LMS was an integral part of 9ECM. Professor Jens Marklof, LMS President, said: "The LMS wanted to demonstrate the commitment of British mathematics to the European mathematics community through a strong presence throughout the 9ECM. It was great to renew relationships and make new contacts for the Society's work. We also sent a full delegation to the EMS Council meeting that preceded the 9ECM to ensure our contribution to the wider work of the EMS." In addition to Marklof, the LMS was represented by Simon Buckmaster (Head of Academic Publications), Simon Edwards (Executive Secretary), Jennifer Gunn (Head of Society Business) and Jason Lotay (Council Member-at-Large).



The LMS stand saw lively traffic throughout, with over 80 mathematicians competing to guess the number of sweets (288) in an LMS sweet jar and many more enquiring about membership, grants and publications. The LMS reception was a lively opportunity for making new friends and meeting old ones. Our special guests were some of the prize winners, namely Juan González-Meneses and Isabel Fernández (President and Vice-President of the ECM organising committee) and former President of the EMS Jean-Pierre Bourguignon.

Figure 2. Attendees at the LMS reception

It is a tradition for the LMS to host a special lecture at each ECM. This year the LMS talk was given by Heather Harrington (University of Oxford, Max Planck Institute of Molecular Cell Biology and Genetics). Along with Wigderson's and Bridson's talks, this was a favourite, opening the door on the field of *topological data analysis*. Harrington explained how, given some complex data, you can construct a *bar code* (a collection of intervals) that encodes important topological information about the data. Such bar codes provide data amenable to statistical analysis and the application of machine learning.

There are many uses of topological data analysis, but Harrington is particularly inspired by applications from biology, such as understanding the structure of proteins or gaining information about diseases from the presence of particular cells in medical tests. Topological data analysis, Harrington says, provides a two-way interchange between maths and biology. Not only is mathematics a useful tool in biology, but biology also generates deep mathematics that is interesting in its own right.

Everyone appeared to very much enjoy the five days in Seville, but for the British contingent there was a tiny tinge of disappointment. On the weekend preceding the ECM, the Council of the European Mathematical Society met in Granada. Members of the Executive Committee were elected, including Victoria Gould of the University of York, who was confirmed as one of the Council's two Vice-Presidents. This was good news. The sadness stems from the fact that London's bid to host the next ECM in 2028 lost out to the Italian city of Bologna. But still, travelling to exciting places is a fun aspect of international conferences that many British mathematicians will no doubt enjoy in four years' time. With some luck I, will write another postcard in 2028, this time from Bologna!

Figure 3. The 9ECM closing ceremony

A podcast featuring interviews recorded at the ECM will appear on plus.maths.org after the summer.

Marianne Freiberger

Marianne Freiberger is the editor of plus.maths.org together with Rachel Thomas. Marianne and Rachel work with researchers to communicate their mathematics to a variety of audiences. They work as part of the Millennium Mathematics Project, based at the University of Cambridge. Apart from editing plus.maths.org, Marianne and Rachel have written and edited several popular science books, worked with local and national radio, TV and streaming media, and various newspapers and magazines. Between them they have over 35 years of experience writing about mathematics for a general audience. After studying semigroup theory at the University of Western Australia, Rachel was a maths consultant working for government and industry. Marianne did her PhD in complex dynamics at Queen Mary, University of London.

Notes of a Numerical Analyst

PDEs and integrals

NICK TREFETHEN FRS

Last spring I taught a course on PDEs. A theme struck me as I collected my thoughts towards the end of the course. PDEs are defined by differentiation, but how often in analysing them we make use of integrals!

An example is the theory of *pseudo-differential operators* (and its cousin microlocal analysis). In one dimension for simplicity, we can write a function as an integral of its Fourier transform:

$$u(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{ik(x-y)}u(y)\,dy\,dk. \qquad (1)$$

Since the derivative of e^{ikx} is $(ik)e^{ikx}$, it follows that u' can be written

$$u'(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} ik\,e^{ik(x-y)}u(y)\,dy\,dk. \qquad (2)$$

Similarly, u'' corresponds to multiplication by $(ik)^2$, and so on. In other words, applying a constant-coefficient linear differential operator to a function u corresponds to multiplying its Fourier transform $\hat{u}(k)$ by a polynomial $p(k)$, which is called the *symbol*. But now the magic comes from allowing p to be more general than a polynomial. If $p(k) = (ik)^{1/2}$, we have taken 'half a derivative' (omitting technical details). And we can let the symbol depend on x too, so a pseudo-differential operator is defined by

$$Lu(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(x,k)\,e^{ik(x-y)}u(y)\,dy\,dk. \qquad (3)$$

Remarkably, differentiation has become a special case of integration. Note that pseudo-differential operators are usually non-local.

Water waves — say, ripples on a pond — illustrate these ideas. Approximately speaking, they are governed by an equation $\partial u/\partial t = Lu$ where L is not a differential but a pseudo-differential operator with symbol $p(k) = i|k|^{1/2}$. We can see the physics of the non-locality by noting that a stationary flat patch of surface may accelerate upward because it is pushed from below by the pressure due to a higher surface elsewhere.

Figure 1. Water waves are governed by an integral rather than differential operator, related to the notion of "half a derivative." (Photo from iStock.)

Another example of integrals at the heart of PDE theory is the *theory of distributions*. If a function is smooth, we can define it pointwise, but we lose smoothness as we take derivatives. How can we rigorously define the Dirac delta function, for example, which should in some sense be the derivative of a step function? The answers come from integrals. A distribution u is defined as a linear functional acting on C^∞ test functions ϕ with compact support: $u : \phi \mapsto (u,\phi)$. When u is an ordinary function, the functional is just $(u,\phi) = \int_{-\infty}^{\infty} u(x)\phi(x)\,dx$. Integration by parts gives $u' : \phi \mapsto -(u,\phi')$, $u'' : \phi \mapsto (u,\phi'')$ and so on. We, thus, find that, viewed as a distribution, every function is infinitely differentiable.

From distributions, it is a small step to *weak solutions* of PDEs, a standard tool of PDE theory and practice. In the end, the very definition of PDE problems thereby comes down to integrals. Perhaps this brings the science full circle, since PDEs are so often derived by taking limits $\Delta x \to 0$ of conservation and balance laws expressed in integral form.

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Mathematics News Flash

Jonathan Fraser reports on some recent breakthroughs in mathematics.

New large value estimates for Dirichlet polynomials

AUTHORS: Larry Guth, James Maynard
ACCESS: arxiv.org/abs/2405.20552

A *Dirichlet polynomial* is a trigonometric sum of the form

$$D(t) = \sum_{n=N}^{2N} b_n e^{it\log n}$$

where N is a positive integer and $\{b_n\}_{n\in[N,2N]}$ is a sequence of complex numbers. It is a key problem in analytic number theory to determine how often $D(t)$ can be 'large'. More precisely, suppose $|b_n| \leq 1$ for all n and $W \subseteq [0,T]$ is a 1-separated set of points t satisfying $|D(t)| > N^\sigma$. Then the problem is to determine how big W can be in terms of N, T and σ.

One can use orthogonality to establish the basic estimate

$$\#W \leq CTN^{1-2\sigma}$$

for all $T \geq N$ for some constant C. The reader may wish to try to prove this for themselves. *Montgomery's large value conjecture* suggests that for $\sigma > 1/2$, the estimate

$$\#W \leq C_\sigma T^{o(1)} N^{2-2\sigma}$$

should hold, where C_σ is a constant now depending on σ. The conjecture remains open in general, and there are many examples where it is known that $\#W \geq cN^{2-2\sigma}$ for some constant c.

This paper, appearing on arXiv in May 2024, improves on the state of the art by providing better estimates in certain important cases, notably when σ is close to $3/4$. These results have several interesting applications, for example, in counting zeroes of the Riemann zeta function lying in certain domains and in counting primes lying in certain 'short intervals.'

I had the privilege of hearing Larry Guth present this work at *Madison Lectures in Harmonic Analysis* two weeks before the paper appeared on arXiv.

A counterexample to the periodic tiling conjecture

AUTHORS: Rachel Greenfeld, Terence Tao
ACCESS: arxiv.org/abs/2211.15847

The periodic tiling conjecture is a famous and fundamental problem in tiling theory. Roughly speaking, it states that any finite subset of the d-dimensional integer lattice $\mathbb{Z}^d$ whose translates can partition $\mathbb{Z}^d$ must also be able to do this with a choice of translates that 'repeats itself.' More precisely, a finite set $T \subseteq \mathbb{Z}^d$ *tiles* $\mathbb{Z}^d$ *by translation* if it is possible to find an infinite collection of translations $\{t_i\}_i$ such that

$$\mathbb{Z}^d = \bigcup_i (T + t_i)$$

with the sets $\{T + t_i\}_i$ pairwise disjoint. For example, T could be the cube $\{0,1\}^d$, and then the set of translations can simply be $(2\mathbb{Z})^d$. This is also an example of a *periodic tiling*, since the tiling is invariant under translations from a d-dimensional lattice. More generally, a tiling by translation is called *periodic* if the set of translations is the finite union of cosets of a finite index subgroup of $\mathbb{Z}^d$. The *periodic tiling conjecture* asserts that if a finite tile T admits a tiling by translation, then it must admit a periodic tiling by translation. This conjecture is true when $d = 1$ (Newman, 1977) and when $d = 2$ (proved by Bhattacharya in 2020 using ergodic theory).

This breakthrough paper, published in *Annals of Mathematics* in 2024, disproves the conjecture for sufficiently large d. Their proof is inspired by a (very complicated) Sudoku-type puzzle.

I tell my hyperbolic geometry students that a tiling is just like a jigsaw: the whole picture must appear; and the jigsaw pieces must not overlap. I also tell my kids that a jigsaw is just like a tiling, but this turns out to be a less effective analogy.

Jonathan Fraser is a pure mathematician based in St Andrews. He is pictured here with teammates shortly before Scotland exited the Euros. There is always next time!

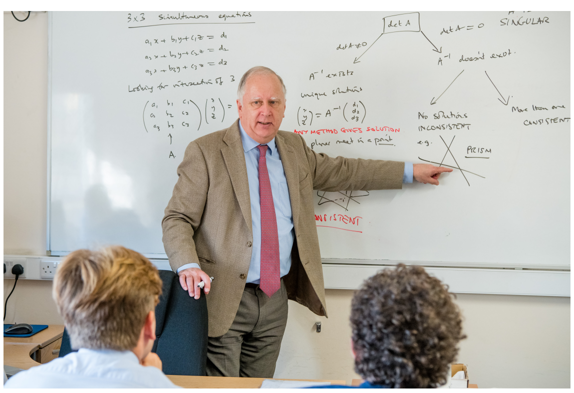
Obituaries of Members

Nicolas J. Lord: 1957–2024

Mr Nick Lord, who was elected a member of the LMS on 20 May 1988, died on 1 March 2024, aged 66. Nick was a senior member of the Society.

Ian Jackson writes: Teacher of mathematics Nick joined Hertford College, Oxford, in 1976 after winning an open scholarship to read mathematics. His interview, however, was not without incident. Interviewees arrived the night before, when they were given a set of problems to work through. Nick confidently told his interviewer that he had completed all but one. When pressed, he explained that the elusive question was insoluble, as it contained a mistake. That would not have been surprising to those who knew him, as he was correct!

He graduated from Hertford with a 1st and a junior maths prize for coming second in his final exams. In the autumn of 1979, he entered Wolfson College, Oxford, having won a senior scholarship to pursue a DPhil specialising in functional analysis. Sadly, when he was well on with his thesis, he discovered that his results had already been published, in Russian. Unable to afford to start afresh, Nick embarked on a PGCE course, his dream of a career at Oxford no longer a reality.



He taught at Tonbridge for over 40 years, and their gain was Oxford's loss. Nick was an inspirational teacher whose lessons were full of humour. Numerous past pupils put their love of maths down to Nick's teaching, including those at the bottom of the ability range right to those at the top. The photograph shows a typical lesson, with 'chalk and talk' done superbly and Nick wearing a typical smile and the same jacket that he wore for all 40 years of teaching.

He was involved for some time as a teacher representative on National Committees considering the future of maths education. He also took part in the earlier days of the UKMT, which supported the mathematical education of the top students in the UK. He went for a number of years to their Trinity Camp for the top 20 in the country, who were competing hard for one of six places in the UK Team. It was a great joy for Nick when towards the end of his career, Tonbridge finally had students who made UK teams.

Nick wrote maths papers throughout his time at Tonbridge having over 200 published by the time of his death, a record it is thought for a practising schoolmaster. These were published widely, but mainly in the *Mathematical Gazette*. He won an award for his brief paper considering the obvious fact that

$$2 = \left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}}.$$

He used that to find an easy example that an irrational to the power of an irrational can be rational. Here the term in brackets is either rational or irrational and in either case the result follows. Two papers that were particularly special for him were those written jointly with current Tonbridge pupils, one to be published later this year. For many years he edited the *Maths Gazette* problem corner. Picking the correct ideas of an appropriate standard for the problems required a wide knowledge. There also followed many interesting correspondences with people around the world who proposed these or attempted their solution. Finally, he needed to synthesise the various solutions into succinct final thoughts on the problem, which always made readers feel that they could (and should) have got there themselves.

A mathematician and teacher to the end. He taught his final lessons less than eight days before his untimely death from an aggressive bowel cancer.

Death Notices

We regret to announce the following death:

- Professor Nelson M. Stephens, who was elected an LMS Member on 19 May 1966, died on 8 January 2024, aged 83.

Metastability and Tipping in Complex Systems

Location: University of Leicester, Leicester, UK
Date: 3–5 September 2024
Website: tinyurl.com/2auh79s2

In September, the University of Leicester will host the international conference *Metastability and Tipping in Complex Systems*. This event is partially supported by the LMS. We invite the submission of abstracts for oral and poster presentations. The participation of graduate students and early career researchers is particularly encouraged.

Dualities in Probability and Algebra

Location: Lancaster University
Date: 16–17 September 2024
Website: tinyurl.com/ye2hp7h6

This two-day workshop brings together experts in probability and algebra to discuss topics at the interface of both disciplines. In particular, several of the talks will explore potential relations between interacting particle systems, representation theory and combinatorics. Mathematicians of all career stages are welcome to attend. The organisers are grateful for partial support from an LMS Celebrating New Appointments (Scheme 9) grant.

LMS/IMA Joint Meeting 2024: Mathematics for the Environment

Location: London and online via Zoom
Date: 20 September 2024
Website: tinyurl.com/lms-ima-24

The LMS is delighted to announce the *2024 LMS/IMA Joint Meeting on Mathematics for the Environment*. We welcome all who are interested to attend this event, with registration open to both LMS and IMA members and non-members. The organising committee members are Victor Shrira (Keele University), Emiko Dupont (University of Bath) and Francisco de Melo Viríssimo (LSE). A full programme and link to register is included on the website.

Applied Mathematics at Sussex

Location: University of Sussex
Date: 10–12 September 2024
Website: tinyurl.com/bdscntrw

This event focuses on the recent developments in applied mathematics and is aimed at bringing together researchers from within the UK who are interested in shape optimisation, infectious diseases and cancer modelling. The event is supported by an LMS Scheme 9 Grant (Celebrating New Appointments) as well as the University of Sussex.

Singularities and Stability in Fluid Dynamics and Kinetic Theory

Location: University of Bath
Date: 18 September 2024
Website: tinyurl.com/39njydwe

This one-day workshop is on the analysis of non-linear partial differential equations in fluids and kinetic theory, focusing on singular solutions, long-time asymptotics and stability properties. Deadline for registration is 13 September 2024. The workshop is funded by the LMS Celebrating New Appointments scheme and the University of Bath.

Liverpool Discrete Maths Colloquium

Location: University of Liverpool
Date: 12–13 November 2024
Website: tinyurl.com/5dv54w6f

This two-day colloquium aims to bring together researchers in discrete mathematics, with topics ranging from structural and algorithmic graph theory to probability and extremal combinatorics.

There will be eight invited lectures by leading researchers representing the topics above. For the benefit of junior researchers, a four-hour mini-course will be given by Maksim Zhukovskii, and there will be a session with short pitches contributed by junior researchers. This event is funded by an LMS Conference Grant.

LMS Meeting

Bookshop Talks 2024

18 September and 16 October 2024, Waterstones Trafalgar Square

Website: tinyurl.com/bookshop-events-24

We are proud to be partnering with Waterstones Trafalgar Square on a new series of events that brings maths ideas to a popular audience. Each event features a conversation between the chair and an author, an audience Q&A and the chance to meet the speakers and get your book signed. The first event in this series took place on Wednesday, 17 July, and featured Sarah Hart in conversation with Rob Eastaway for *An Evening of Mathematics and Literature*. The second two events will feature David Spiegelhalter in conversation with Timandra Harkness on *The Art of Uncertainty* (18 September) and Marcus du Sautoy with a chair to be confirmed on *Around the World in 80 Games: A Mathematician Unlocks the Secrets of the Greatest Games*.

LMS Joint Meeting

Black Heroes of Mathematics Conference 2024

2–3 October 2024, De Morgan House, London, and online via Zoom

Website: tinyurl.com/black-heroes-24

The vision of this conference is to celebrate the inspirational contributions of black role models to mathematics and mathematics education. There will be a balance of technical talks by internationally renowned speakers and, to provide a testimonial dimension, some details of career paths and their experience. Confirmed speakers include Angela Tabiri, Robin Wilson, Teresa Senyah, Justice Moses Kwaku Aheto, Imoleayomide Ajayi, Flavia Santos, Nathalie Ayi and Kim Sellers.

LMS Meeting

LMS Annual General Meeting and Naylor Lecture

22 November 2024, BMA House, London, and online via Zoom

Website: tinyurl.com/lms-agm-24

This year's Annual General Meeting will be held at BMA House, London, on 22 November 2024. The meeting will begin with Society Business, including the announcement of the LMS election results and presentation of the 2024 LMS Prizes. This will be followed by the Naylor Lecture, to be given by the 2023 Naylor Prize and Lectureship winner Professor Jens Eggers (Bristol). A supporting lecture will be given by Professor Marco Fontelos (Madrid). A reception and the Annual Dinner will follow the event. See the website for details and to register.

Society Meetings and Events

September

18 Bookshop Talk, Waterstones Trafalgar Square (512)
20 LMS/IMA Joint Meeting 2024, London and online via Zoom (512)

October

2–3 Black Heroes of Mathematics Conference, De Morgan House, London, and online via Zoom (512)
16 Bookshop Talk, Waterstones Trafalgar Square (512)

November

22 LMS Annual General Meeting and Naylor Lecture, BMA House, London, and online via Zoom (512)

Calendar of Events

This calendar lists Society meetings and other mathematical events. Further information may be obtained from the appropriate *LMS Newsletter* whose number is given in brackets. A fuller list is given on the Society's website (lms.ac.uk/content/calendar). Please send updates and corrections to calendar@lms.ac.uk.

September

3–5 Metastability and Tipping in Complex Systems, University of Leicester (512)
10-12 Applied Mathematics at Sussex, University of Sussex
16–17 Dualities in Probability and Algebra, Lancaster University (512)
18 Singularities and Stability in Fluid Dynamics and Kinetic Theory, University of Bath (512)

November

12–13 Liverpool Discrete Maths Colloquium, University of Liverpool (512)