LONDON MATHEMATICAL SOCIETY
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NEWSLETTER

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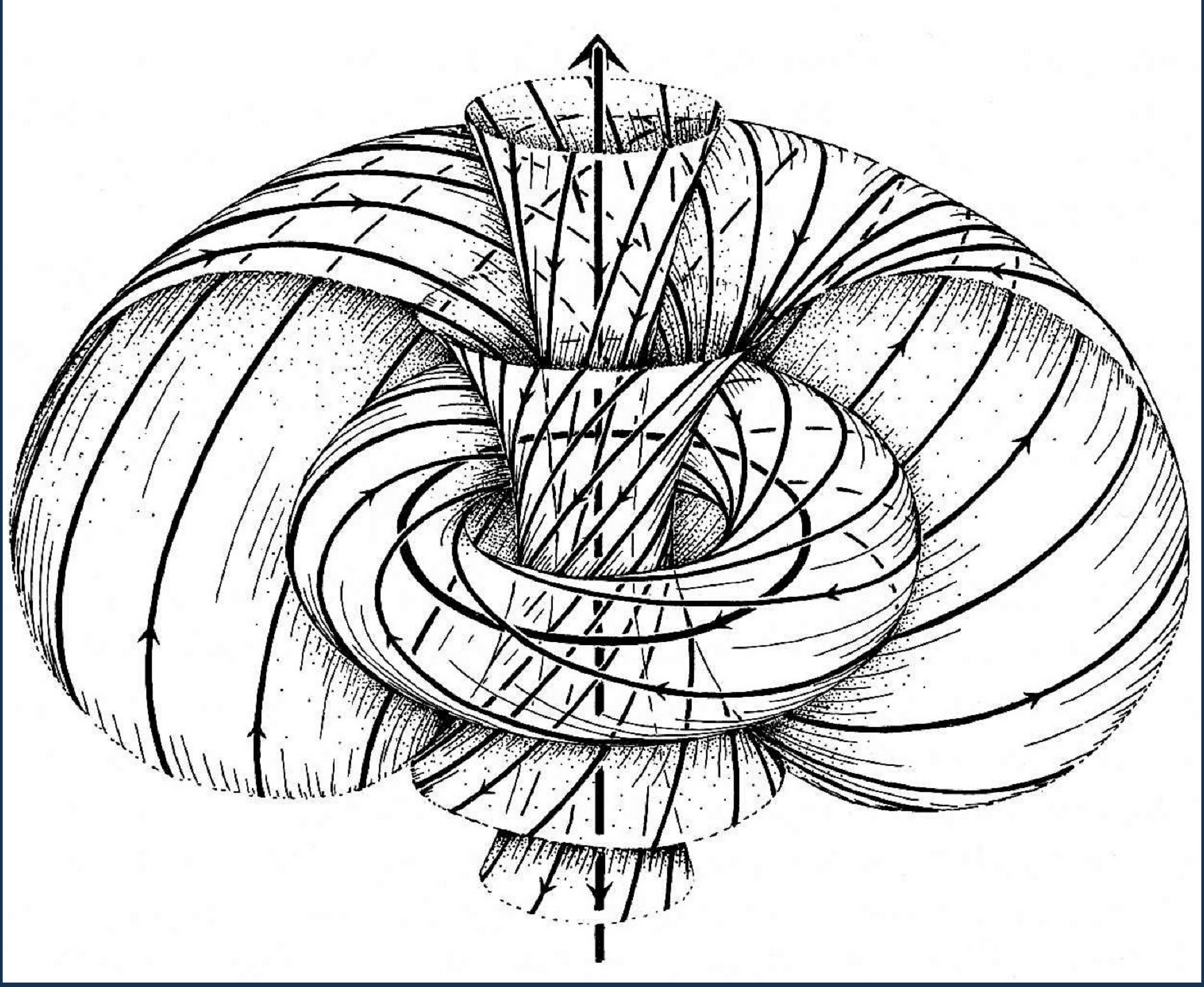

ROGER PENROSE: A SPEAKER AND HIS AUDIENCE

INTEGRATION IN ELEMENTARY TERMS

AN INTERVIEW WITH SIMON TAVARÉ

SUBMISSIONS

The Newsletter welcomes submissions of feature content, including mathematical articles, career related articles, and micro-theses from members and non-members. Submission guidelines and LaTeX templates can be found on newsletter.lms.ac.uk/submissions.

Feature content should be submitted to the editor-in-chief at iain.moffatt@rhul.ac.uk.

News items should be sent to
newsletter@lms.ac.uk.

Notices of events should be prepared using the templates on newsletter.lms.ac.uk/submissions and sent to calendar@lms.ac.uk.

For advertising rates and guidelines see newsletter.lms.ac.uk/rate-card.

NEWSLETTER WEBSITE

The Newsletter is freely available electronically through the LMS's website newsletter.lms.ac.uk.

COVER IMAGE

Courtesy of Roger Penrose. See page 23.

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CONTENTS

NEWS 4–16	The latest from the LMS and elsewhere	4
LMS BUSINESS 17–20	Reports from the LMS	17
FEATURES 21–44	Roger Penrose in Warsaw: a Speaker and His Audience	21
	Integration in Elementary Terms	30
	An Interview with Simon Tavaré	37
	The Danish Mathematical Society	43
	Member Consultation: Broadening Support for Diversity in Mathematics	44
EARLY CAREER 45–50	Success Stories in Mathematics	45
	Microthesis: Mathematical Models for Glass Sheet Manufacture	46
	Applying for Your First Grant	48
REVIEWS 51–55	Books and events	51
OBITUARIES 56–57		56
EVENTS 58–62	Latest announcements	58
CALENDAR 62–64	All upcoming events	62

IN BRIEF

Somerville Proves Noteworthy

Ross McEwan (Chief Executive of RBS), Brigitte Stenhouse, Malcolm Buchanan (Chair of RBS Scotland Board)

On 4 October 2017 The Royal Bank of Scotland launched their new polymer £10 note, featuring Scottish mathematician and expositor Mary Somerville.

Somerville was an expert in French analysis in the early 19th Century, and her translation of Laplace's work *Mechanism of the Heavens* helped disseminate these ideas to England. Somerville's portrait features on the front of the note beside an extract from her 1834 publication *On the Connexion of the Physical Sciences*: "Anyone who has observed the reflection of the waves from a wall on the side of a river, after the passage of a steamboat, will have a perfect idea of the reflection of sound and light." In the background is an image of Burntisland, where in her youth she conducted solitary investigations of the local plant and sea life.

The polymer note, which is 15% smaller than the current version and includes braille, features one of Somerville's diagrams of the motion of the moon around the Earth, which appears only under UV light. RBS chief executive Ross McEwan, acknowledging the impact Somerville's work had on our understanding of the world, said, "it is fitting that our most advanced note yet will carry her portrait".

The RBS £5 polymer note, featuring Scottish novelist Nan Shepherd, was released into circulation in 2016. The final note in this series will feature Glasgow entrepreneur Catherine Cranston.

Brigitte Stenhouse
The Open University

LMS Publications News

Mandating ORCID from 2018

ORCID (Open Researcher and Contributor ID) is a not-for-profit organisation that provides unique identifiers for researchers engaged in academic publishing and scholarship. An ORCID iD distinguishes researchers from others, simplifies the searching of databases and indexes, and supports manuscript submissions and grant applications. As such it saves time, connects researchers with funders, societies and other institutions, and creates a persistent thread to link research activities. The LMS is joining many other academic publishers and funding bodies in making ORCID iDs required as part of the submission process to its journals.

From 2018, ORCID iDs will be mandatory for all corresponding authors submitting a paper to the *Bulletin, Journal, Proceedings* and *Transactions of the London Mathematical Society*, and to the *Journal of Topology*. Corresponding authors will be asked to provide their ORCID iD as part of the submission process; those without an ORCID iD will be shown how to obtain one. This can be done in just a minute or two via the website orcid.org.

Content Sharing: New Peer-to-Peer Feature Launched by Wiley

As of July 2017, subscribers of the *Bulletin, Journal* and *Proceedings of the LMS*, and the *Journal of Topology*, are able to share content with each other in a new way.

Our publishing partner Wiley has introduced a 'peer-to-peer' sharing initiative on their Online Library, where articles published in these journals are accessed digitally. Subscribers (with existing full-text access to the journals) can generate a URL that can be shared with other readers in the form of an ePDF provided by academic paper manager ReadCube.

When shared with a fellow subscriber, the URL provides an unrestricted view of the electronic PDF and the usual ability to download or print it; a non-subscriber is granted restricted access to a read-only PDF with no print or download privileges.

This new way to share articles should benefit researchers, institutions and society as a whole, facilitating collaboration and hopefully achieving a wider readership and impact of research.

***Transactions of the LMS* now listed in the Directory of Open Access Journals**

Launched in 2013 and published by Wiley on behalf of the LMS, the *Transactions of the LMS* is the Society's fully open access, online journal. As of July 2017, the journal has been listed in the Directory of Open Access Journals (DOAJ).

DOAJ is an independent, community-curated online directory that indexes and provides access to quality open access, peer-reviewed journals. Currently containing the information of over 9,000 publications, DOAJ aims to increase the ease of use of those open access academic journals that adhere to criteria for quality control and best practice.

All content published in *Transactions of the LMS* can be accessed for free online at bit.ly/TLMSlms. Inclusion in DOAJ signifies *Transactions of the LMS* as a quality open access journal, and increases the visibility and impact of the mathematical papers that it publishes.

John Hunton
LMS Publications Secretary

PEOPLE

LMS Hardy Lectureship 2018

LAUREN WILLIAMS

The London Mathematical Society is pleased to announce the LMS Hardy Fellow 2018 is Professor Lauren Williams (UC Berkeley).

The Hardy Fellowship was founded in 1967 in memory of G.H. Hardy in recognition of outstanding contribution to both mathematics and to the Society. The Hardy Fellowship is a lecture tour of the UK by a mathematician with a high reputation in research.

Professor Williams will undertake a lecture tour of the UK from 29 June – 13 July 2018, which will start with the Hardy Lecture at the Society Meeting on Friday 29 June in London. Professor Williams' research is highly interdisciplinary and has had a broad impact on several areas of mathematics, ranging from combinatorics and algebra to probability and mathematical physics, with applications to particle processes and shallow water waves.

Professor Williams delivers excellent talks in which she complements clarity with really advanced inspiring mathematics. As a down-to-earth and engaging speaker, Williams' lectures will illustrate how combinatorial tools can be very useful in algebra, probability, integrable systems, and mathematical physics.

Further details about the Hardy Lecture Tour 2018 will be announced in the New Year and available on the website lms.ac.uk/events/lectures/hardy-lectureship.

LMS Invited Lecture Series 2018

ART OWEN

Art Owen is a professor of Statistics at Stanford University. He is a long-standing elected fellow of both the ASA (American Statistical Association) and the Institute for Mathematical Statistics (IMS). In addition, he has a number of other accolades, including the prestigious IMS Medallion Lecture.

Owen is one of the world's leading researchers in Monte Carlo methods (statistical algorithms which use the simulation of random processes to draw inference on quantities of interest) and, in particular, quasi-Monte Carlo (QMC) methods. Owen introduced a randomised version of QMC that allows error estimates via replicates, further reducing the error on quantities of interest over vanilla Monte Carlo (and QMC) approaches. His work on QMC has found practical application in financial valuation and computer graphics, among many other areas. Owen's best known work is on empirical likelihood and its uses in statistical inference, including in econometrics and survival analysis. Owen has also made contributions to bioinformatics, and especially the genomics of ageing.

Professor Owen will give the LMS Invited Lecture Series 2018 on *From the Foundations of Simulation to Quasi Monte Carlo* in the Zeeman Building of the

University of Warwick from the 9-13 July 2018. He will be accompanied by supporting lectures from Professor Nicolas Chopin (ENSAE) on *Sequential Monte Carlo*, Professor Mark Huber on *Perfect Simulation* and Professor Jeff Rosenthal (Toronto) on *Optimising and Adapting Metropolis*. Full details can be found at tinyurl.com/y9kda8jw.

The annual Invited Lecturers scheme aims to bring a distinguished overseas mathematician to the United Kingdom to present a small course of about ten lectures spread over a week. Each course of Invited Lectures is on a major field of current mathematical research, and is instructional in nature, being directed both at graduate students beginning research and at established mathematicians who wish to learn about a field outside their own research specialism.

Savilian Professorship

FRANCES KIRWAN

Past LMS President Professor Dame Frances Kirwan FRS has been elected to the Savilian Professorship at the University of Oxford. Professor Kirwan is the first woman to hold the Professorship since it was founded in 1619.

Throughout her notable career Professor Kirwan has made many outstanding contributions in the area of algebraic geometry. During her career she has also received many honours including being elected a Fellow of the Royal Society in 2001 (only the third female mathematician to attain this honour), and the President of the London Mathematical Society from 2003-2005 (the second female ever elected). Professor Kirwan has for many years been active in addressing the gender imbalance in mathematics, both as a past member of the LMS Women in Mathematics Committee and also in European Women in Mathematics, having served as Convenor, as well as chairing the Prizes Committee of the European Mathematical Society's 6th European Congress of Mathematicians in 2013, the year she was also awarded an LMS Senior Whitehead Prize

2017 Cecil King Travel Scholarship

SCOTT HARPER

The 2017 Cecil King Travel Scholarship has been awarded to Scott Harper (University of Bristol). Each year the London Mathematical Society awards this £5,000 scholarship to a young mathematician of outstanding promise to support a period of study or research in mathematics abroad.

Scott is currently a third-year PhD student, under the supervision of Dr Tim Burness. In spring 2018, Scott will visit the University of Western Australia, where he will work with Professor Michael Giudici, and the University of Auckland, where he will work with Professor Marston Conder and Professor Eamonn O'Brien. At both institutions, Scott will work on problems at the interface between group theory and combinatorics, such as questions pertaining to the generating graph of groups.

Generating sets of finite groups, especially finite simple groups, have been a focus of research since the earliest days of group theory. The generating graph is a modern combinatorial tool which captures many natural generation properties and which leads to many striking questions and conjectures on the generation of finite groups.

MATHEMATICS POLICY ROUNDUP

HEFCE Guidance for REF2021

The Higher Education Funding Council for England (HEFCE) has published a document outlining the first set of initial decisions on the Research Excellence Framework. A further set of decisions will be taken on the remaining aspects of the framework in the autumn, incorporating further consultation activity. More information is available at tinyurl.com/y75ukujw.

Mathematics Examination Results

The number of A-level Mathematics entries across the UK is up 3.3% on last year, with 95,244 students sitting the exam. Figures released by the Joint Council for Qualifications also show that A-level Further Mathematics entries have increased by 6.0% (to 16,172). AS Mathematics entries decreased by 1.4% (to 160,450) and AS Further Mathematics entries increasing by 4.6% (to 27,980). Detailed results, including for GCSEs, at tinyurl.com/y82okjbr.

Functional Skills Consultation

The DfE has opened a consultation on its approach to regulating reformed functional skills qualifications in English and Mathematics. More information is available at tinyurl.com/y8e7geaa. The closing date for submissions is 22 November 2017.

Teacher Retention

The Wellcome Trust has published research showing that 'continuing professional development can play a significant role in retaining science teachers and improving science education'. More information is available at tinyurl.com/y6uu8lzu.

Commons Science and Technology Select Committee

The membership of the Committee has been agreed. More information is available at tinyurl.com/y92t4mrz.

John Johnston
Joint Promotion of Mathematics

EUROPEAN

2017 CNRS Gold Medal

A 2017 CNRS Gold Medal has been awarded to Thibault Damour for his essential contributions to the discovery of gravitational waves. After obtaining his doctorate from the University of Paris VI in 1974 Thibault Damour worked as a post-doc at Princeton University (USA). From 1977 to 1989 he was a CNRS researcher and in 1989 was recruited as a permanent professor in theoretical physics at the Institut des Hautes Études Scientifiques (IHES).

Damour is a theoretical physicist working on relativistic gravity (Einstein's theory of General Relativity), cosmology and the extensions to relativistic gravity suggested by string theory. He made innovative contributions to the theory of black holes and to several aspects of primordial cosmology, linking the theory of General Relativity and observations. Most importantly, he developed in 2000 a new method, called Effective One Body (EOB), which for the first time described the complete gravitational signal emitted by the coalescence of two black holes. This analytical approach (further enriched by the results of numerical simulations) was used by the LIGO-Virgo collaboration to subtract the noise and analyse in terms of physical parameters (mass, spin) the gravitational waves detected since September 2015. The EOB method has been extended to the description of the gravitational signal emitted by the coalescence of binary neutron stars until they become so close that they collide. This precise theoretical description could allow information on the equation of state of nuclear matter to be obtained from the gravitational signal.

This year the CNRS has awarded two Gold Medals: one to Damour and the other to Alain Brillet, cited as a "visionary in the development of gravitational waves detectors, [and] one of the fathers of the European experiment Virgo".

[Edited version of IHES Press Release 27 September 2017.]

ERC Plan for 2018

While the UK remains in the EU it is worth noting the European Research Council's grant competition for 2018 with a budget of around €1.86 billion mostly earmarked for early- to mid-career researchers. The ERC is also reintroducing *Synergy Grants*, the funding scheme for groups of two to four scientists who jointly address ambitious research problems. Carlos Moedas, Commissioner for Research, Science and Innovation, said: "This is the starting whistle for the next round of this champions' league of European research within the EU's Horizon 2020 research and innovation programme". The ERC President Professor Jean-Pierre Bourguignon said: "These grants can trigger unconventional collaborations, allow for the emergence of new fields of study and help put scientists working in Europe at the global forefront. By providing €250 million of funding for the Synergy Grant call, the ERC Scientific Council intends to make possible substantial advances at the frontiers of knowledge which would be impossible for researchers working alone."

The application deadline for Synergy Grants is 14 November 2017. Details at erc.europa.eu/news.

Caucasian Conference

To what extent can mathematicians contribute to peace and harmony in the world? This question was posed when the first Caucasian Mathematics Conference (CMC) was announced in the EMS e-News in 2014. The region was affected by old and new tensions, and unfortunately since then the situation has worsened almost by the day. Despite the difficulties, and after postponing the conference for a year, the second CMC was held in Turkey, at the Yüzüncü Yıl University of Van, 22-24 August 2017. It was held under the auspices of the EMS, with the cooperation of the Armenian, Azerbaijan, Georgian, Iranian, Russian and Turkish Mathematical Societies. The CMC's primary aim is to bring together mathematicians from Caucasian and neighbouring countries biannually in one of these countries. Details at: euro-math-soc.eu/cmc.

Polish Mathematical Society

The 9th edition of *The International Stefan Banach Prize for a Doctoral Dissertation in the Mathematical Science* has been awarded to Anna Szymusiak (Jagiellonian University, Kraków, Poland). This prize is awarded jointly by Ericsson and the Polish Mathematical Society. Its aim is to promote and financially support the most talented young researchers in mathematical sciences from central and north Europe. Details at: banachprize.org.

Portuguese Mathematical Society

Awarded this year for the first time, the prestigious *Gulbenkian Prize – Knowledge* for the Promotion of Educational Success (value €50,000) was won by the Portuguese Mathematical Society for its creation and organization of the Portuguese Mathematics Olympiad, which this year celebrated its 35th edition. The jury praised the Olympiad as an "educational initiative of reference and great national impact, which for more than three decades has promoted enjoyment of the discipline", and for connecting tens of thousands of students from primary and secondary schools with mathematics "in a stimulating and creative environment".

David Chillingworth
LMS/EMS Correspondent

Heilbronn Institute for Mathematical Research

Heilbronn Research Fellowships

Salary: £36,613-£41,212
3 years fixed term, full-time

The School of Mathematics at the University of Bristol invites applications for one or more Research Fellowships in association with the Heilbronn Institute for Mathematical Research. Research fellows divide their time equally between their own research and the research programme of the Heilbronn Institute, which offers opportunities to engage in collaborative work as well as individual projects.

Research areas of interest include but are not restricted to Algebra, Algebraic Geometry, Combinatorics, Computational Statistics, Data Science, Number Theory, Probability, and Quantum Information. These areas are interpreted broadly: Fellows have previously been appointed with backgrounds in most areas of Pure Mathematics and Statistics, and in several areas of Mathematical/Theoretical Physics.

For more information about the Heilbronn Institute, see **http://heilbronn.ac.uk**.

Due to the nature of the Heilbronn Institute's work, you must satisfy vetting before appointment. UK resident UK nationals will normally be able to meet this condition: other potential applicants should consult the Heilbronn Manager at **heilbronn-manager@bristol.ac.uk** about their eligibility before applying. You may become a member of the USS pension scheme.

There is a salary supplement of £3.5K pa, in recognition of the distinctive nature of these Fellowships. Payment of this supplement is conditional on a finished thesis having been accepted in final form, because we expect Heilbronn Fellows to hold PhDs before working at the Heilbronn Institute. In addition, a fund of at least £2,000 per annum to pay for research expenses will be available to each Fellow.

The Fellowship will be for three years, with a preferred start date in October 2018, though another date may be possible by prior agreement.

The School of Mathematics is a supporter of the LMS Good Practice Scheme aiming at advancing women's careers in mathematics and we particularly welcome applications from women for this post. Candidates interested in learning more about the working environment at the Institute prior to application are welcome to contact the Associate Chair, Dr Julia Wolf, at **assoc-chair-himr@bristol.ac.uk**.

For more information about the application procedure, please visit our website: **http://heilbronn.ac.uk/opportunities**.

The application deadline is 9am on Friday the 24th of November 2018.

University of BRISTOL

OPPORTUNITIES

LMS Prizes 2018: call for nominations

The LMS would like to invite nominations for the following prizes in 2018, which are intended to recognise and celebrate achievements in and contributions to mathematics:

The **Pólya Prize** in recognition of outstanding creativity in, imaginative exposition of, or distinguished contribution to, mathematics within the UK; the **Fröhlich Prize** for original and extremely innovative work in any branch of mathematics; the **Senior Berwick Prize** for an outstanding piece of mathematical research published by the Society during the eight years ending on 31 December 2017; the **Whitehead Prizes** for work in and influence on mathematics (up to six may be awarded); the **Hirst Prize and Lectureship** for contributions to the study of the history of mathematics; and the **Anne Bennett Prize** for work in and influence on mathematics, particularly to women mathematicians.

Regulations and nominating forms can be found at tinyurl.com/lmsprizes. Please return nominating forms to Katherine Wright, Society Business Officer: prizes@lms.ac.uk.

The closing date for nominations is 26 January 2018. Any nominations received after that date will be considered in the next prize award round.

Christopher Zeeman Medal 2018: call for nominations

The Councils of the LMS and the IMA are inviting nominations for the 2018 award of the Christopher Zeeman Medal, which is the UK award dedicated to recognising excellence in the communication of mathematics.

The IMA and the LMS wish to honour mathematicians who have excelled in promoting mathematics and engaging with the general public. They may be academic mathematicians based in universities, mathematics school teachers, industrial mathematicians, those working in the financial sector or indeed mathematicians from any number of other fields. Most importantly, these mathematicians will have worked exceptionally to bring mathematics to a non-specialist audience, whether it is through giving public lectures, writing books, appearing on radio or television, organising events or through an entirely separate medium.

A form for nominations is available at tinyurl.com/zeemanmedal.

Please email any enquiries to Katherine Wright, Society Business Officer, London Mathematical Society: prizes@lms.ac.uk.

Nominations must be received by 28 February 2018.

Louis Bachelier Prize 2018: call for nominations

The Louis Bachelier Prize is a biennial prize jointly awarded by the London Mathematical Society, the Natixis Foundation for Quantitative Research and the Société de Mathématiques Appliquées et Industrielles. The Prize will be awarded to a mathematician who, on the 1st January of the year of its award, has fewer than 20 years (full time equivalent) of involvement in mathematics at postdoctoral level, allowing for breaks in continuity, or who in the opinion of the Bachelier Prize Committee is at an equivalent stage in their career.

The Prize will be awarded to the winner for his/her exceptional contribution to mathematical modelling in finance, insurance, risk management and/or scientific computing applied to finance and insurance. The prize winner will receive €20,000 including £5,000 to organise a scientific workshop in Europe on their area of research interests.

Nominations are now open for the 2018 Louis Bachelier prize. A form for making nominations is available to download at tinyurl.com/bachelier.

LMS Durham Symposia 2019: call for proposals

The London Mathematical Society invites proposals for Durham Symposia in 2019, and intends to support two Symposia to take place in August 2019.

The Symposia began in 1974, and have now become an established and recognised series of international research meetings. They provide an excellent opportunity to explore an area of research in depth, to learn of new developments, and to instigate links between different branches. The format is designed to allow substantial time for interaction and research. The meetings are by invitation only and held in August, lasting five days, with up to 50 participants, roughly half of whom will come from the UK. They are held at the University of Durham.

Prospective organisers should send a formal proposal to the Durham Representative, Dirk Schuetz (dirk.schuetz@durham.ac.uk) by 20 November 2017.

Proposals should include:

- A full list of proposed participants, divided into specific categories (please see the guidance on submission of proposals at lms.ac.uk/events/durham-symposia for more details). Proposers are encouraged to actively seek to include women speakers and speakers from ethnic minorities, or explain why this is not possible or appropriate.
- A detailed scientific case for the symposium, which shows the topic is active and gives reasons why UK mathematics would benefit from a symposium on the proposed dates.
- Details of additional support from other funding bodies.

The Durham Representative will provide an estimated cost for accommodation for the symposium and estimated travel costs for each participant.

For further details about the Durham Symposia, please visit the Society's website: lms.ac.uk/events/durham-symposia.

Before submitting: Organisers are welcome to discuss informally their ideas with the Durham Representative (dirk.schuetz@durham.ac.uk) and/or the Chair of the Research Grants Committee, Dr Francis Clarke (grants@lms.ac.uk).

CWM funding call for 2018 mathunion.org/cwm

The International Mathematical Union's Committee for Women in Mathematics (CWM) invites proposals for funding of **up to €3,000** for activities taking place in 2018, aimed at either:

(1) Establishing or supporting networks for women in mathematics, preferably at the continental or regional level, and with priority given to networks in developing or emerging countries.

(2) Organising a mathematical school open to all with all women speakers and mainly women organisers. This type of mathematical school, which should include a significant proportion of time devoted to background and introductory material, can be a very effective way of showcasing the contributions of women mathematicians and creating an opportunity for female students to be in touch with women leaders, without excluding male students. Expenses covered by CWM could include, for example, costs for speakers, women organisers, or for women participants.

(3) Other ideas for researching and/or addressing issues encountered by women in mathematics may also be considered.

Proposers should write a short account (no more than two pages) explaining the nature of their activity and how it fulfils one of the above aims, as well as indications on how the CWM money would be spent and other funding which may be available, with deadline 15 December 2017. Applications should be sent to applications-for-cwm@mathunion.org.

Successful applications will be informed no later than 31 January 2018. Details at tinyurl.com/ycmyly5h.

LMS Research Workshop Grants: call for proposals

The LMS offers grants to support Research Workshops held in the UK. Requests for support (for travel and subsistence of participants, and reasonable associated costs) in the range **£1,000-£10,000** will be considered. The maximum award is £10,000, but a typical award is in the range of £3,000-£5,000. Applications for partial support of workshops with other sources of support will be considered.

Research Workshops are an opportunity for a small group of active researchers to work together for a concentrated period on a specialised topic. The primary purpose of the scheme is to support new research initiatives rather than meetings which form part of an established series. Grant requests for conferences should made via a Scheme 1 (Conference Grant) application: lms.ac.uk/grants/conference-grants-scheme-1.

There is no prescribed format for an LMS workshop, but it is expected that the number of participants will be no more than 40, and could be as low as 10, meeting for a period of several days.

All participants should be actively involved in the programme, and should be identified in the proposal; the participation of appropriate postdocs and graduate students is encouraged.

Applications can be made at any time but should normally be submitted at least 12 months before the proposed workshop. All proposals are refereed, and support will only be offered if it felt that the benefits to UK mathematics are likely to be significant.

Applications should be made via email to the Chair of the Research Grants Committee, Dr Francis Clarke (grants@lms.ac.uk) any time. There is no application form. Proposals in PDF or Word format should contain: 1) a description of the research area; 2) the aims and format of the workshop; 3) a list of participants and a budget; and 4) details of proposed location and timing.

Applications will be reviewed by the Research Grants Committee and may be sent out for peer-review.

Applicants may consult Dr Francis Clarke, or Anthony Byrne, the Grants and Membership Administrator (grants@lms.ac.uk) informally about their proposed workshop before making an application.

LMS Invited Lectures Series 2019: call for proposals

The annual LMS Invited Lectures Series consists of meetings held in the UK at which a single speaker gives a course of about ten expository lectures, examining some subject in depth, over a five day period (Monday to Friday) during a University vacation. The meetings are residential and open to all interested.

The LMS Invited Lecturer is offered a **£1,250** honorarium for giving the course plus full expenses. A **£4,000** grant is given to the host department to support attendance at the lectures.

Proposals for the Invited Lectures 2019

Any member who would like to suggest a topic and lecturer and be prepared to organise the meeting at their own institution or a suitable conference centre can submit a proposal. For further details, please visit the Society's website: lms.ac.uk/content/invited-lecturer-proposals

The deadline for proposals is 2 February 2018.

LMS Invited Lecturer 2018

The LMS Invited Lecture Series 2018 will be given by Professor Art Owen (Michigan State University) at Warwick from 9-13 July 2018 (see page 5).

Previous Invited Lecturers can be found at tinyurl.com/yaxwchdr.

Enquiries about the LMS Invited Lectures may be addressed to the LMS Programme Secretary (lmsmeetings@lms.ac.uk).

Spanish Society for Applied Mathematics

The second joint mathematical meeting Spain-Brazil organized by RSME, SeMA, SBM and SBMAC will take place in Cádiz, Spain, from 11-14 December 2018. Those interested in organizing a special session are invited to email their proposal by 15 December 2017 as a PDF file to spa-braz-math-cadiz2018@uca.es. For more information see: tinyurl.com/yaxlo5a9.

Cecil King Travel Scholarship: call for applications

The London Mathematical Society annually awards a **£5,000** Cecil King Travel Scholarship in Mathematics, to a young mathematician of outstanding promise. The Scholarship is to support a period of study or research abroad, typically for a period of three months, in any area of mathematics. Established in 2001 by the Cecil King Memorial Fund, the Cecil King Travel Scholarship award is made by the LMS Council on the recommendation of the Cecil King Prize Committee.

Applicants must be nationals of the UK or the Republic of Ireland and either registered for or having completed a doctoral degree within 12 months of the closing date.

To apply, please complete the application form (lms.ac.uk/prizes/cecil-king-travel-scholarship) and include a written proposal describing the intended programme of study or research abroad, and the benefits to be gained from such a visit. The application deadline is 31 March 2018.

Shortlisted applicants will be invited to interview during which they will be expected to make a short presentation on their proposal. Interviews will take place at the University of Birmingham in May 2018.

Queries may be addressed to Elizabeth Fisher (ecr.grants@lms.ac.uk).

Spitalfields Day 2018: call for proposals

The London Mathematical Society is pleased to offer funding of **up to £1,000** towards the cost of a Spitalfields Day. A Spitalfields Day is a one-day event at which selected participants, often eminent experts form overseas, give survey lectures or talks, which are accessible to a general mathematical audience. The Spitalfields Day is often associated with a long-term symposium and speakers will generally give lectures on topics of the symposium.

The name honours the Society's predecessor, the Spitalfields Mathematical Society, which flourished from 1717 to 1845, and Spitalfields Days have been held each year since 1987.

The funding of £1,000 is intended to cover actual supplementary costs for the event, e.g. subsidising the cost for a lunch for participants, and for small travel grants of £50 to enable LMS members and research students to attend the event.

If you are interested in organising a Spitalfields Day, please write to the Society (lmsmeetings@lms.ac.uk). The format need not be precisely as described, but should be in a similar spirit. The next deadline for proposals is **31 January 2018**. (Subsequent deadline is 15 September 2018. Please note the Society cannot fund events retrospectively so applicants are advised to apply well in advance of the event).

Previously supported Spitalfields Days can be seen on the LMS website at lms.ac.uk/events/spitalfields-days.

LMS Undergraduate Research Bursaries in Mathematics 2018

The Undergraduate Research Bursary scheme provides an opportunity for students in their intermediate years to explore the potential of becoming a researcher. The award provides **£180/£190 per week** to support a student undertaking a 6–8 week research project over Summer 2018, under the direction of a project supervisor.

Students must be registered at a UK institution for the majority of their undergraduate degree, and may only take up the award during the summer vacation between the intermediate years of their course. Students in the final year of their degree intending to undertake a taught Masters degree immediately following their undergraduate degree may also apply. Applications must be made by the project supervisor on behalf of the student.

For further information and to download the application form, visit tinyurl.com/undergradbursaries or search "LMS URB". Queries may also be addressed to George Ross (urb@lms.ac.uk).

The closing date for receipt of applications is 5pm Friday 16 February 2018.

LMS Research Schools 2019: call for proposals

The London Mathematical Society invites proposals for Research Schools to be held in the UK in 2019.

LMS Research Schools

Proposals are invited for LMS Research Schools. **Up to £20,000** is available for a LMS Research School which provides training for research students in all contemporary areas of mathematics. The LMS Research Schools support participation of research students from both the UK and abroad. The lecturers are expected to be international leaders in their field. The LMS Research Schools are often partially funded by the Heilbronn Institute for Mathematical Research.

How to apply

Information on how to submit a proposal and a list of previously supported Research Schools can be found at: lms.ac.uk/events/lms-research-schools.

Applicants are strongly encouraged to discuss their ideas for Research Schools with the Chair of the Early Career Research Committee, Professor Chris Parker (research.schools@lms.ac.uk), before submitting proposals.

Proposals should be submitted to Elizabeth Fisher (research.schools@lms.ac.uk) by **31 January 2018**.

Clay Mathematics Institute Enhancement and Partnership Program

To extend the international reach of the Research School, prospective organisers may also wish to consider applying to the Clay Mathematics Institute (CMI) for additional funding under the CMI's Enhancement and Partnership Program. Further information about this program can be found at:

tinyurl.com/y72byonb. Prospective organisers are advised to discuss applications to this program as early as possible by contacting the CMI President, Nick Woodhouse (president@claymath.org). There is no need to wait for a decision from the LMS on your Research School application before contacting the CMI about funding through this program.

VISITS

Visit of Gabriele Fici

Dr Gabriele Fici (University of Palermo, Italy) will be visiting the Department of Informatics, King's College London from 7 to 14 November 2017. His research focuses on the combinatorial and algorithmic aspects of strings. He is interested in problems on finite and infinite strings arising in different areas of theoretical computer science and discrete mathematics. Dr Fici will give a lecture on 13 November and also at the University of Leicester on 10 November (contact Rajeev Raman: rr29@leicester.ac.uk). For further details contact Kathleen Steinhofel (kathleen.steinhofel@kcl.ac.uk). The visit is supported by an LMS Scheme 2 grant.

Visit of Wafaa Batat

Dr Wafaa Batat from the Ecole Polytechnique d'Oran will visit Newcastle University from 4 to 22 December 2017. She is an early career researcher working on understanding and classifying geometric objects on manifolds with large symmetry groups. She is particularly interested in Einstein metrics, Ricci solitons and distinguished submanifolds.

For information regarding Dr Batat's visit contact Stuart Hall (stuart.hall@ncl.ac.uk). The visit is supported by an LMS Scheme 4 (research in pairs) grant.

Visit of Michele Zordan

Dr Michele Zordan (KU Leuven) will visit Durham University from 7 to 21 January 2018. Zordan is working on representation zeta functions and related techniques from p-adic integration and model theory. For information regarding Dr Zordan's visit contact Alexander Stasinski (alexander.stasinski@durham.ac.uk). The visit is supported by an LMS Scheme 4 (research in pairs) grant.

Visit of David Hilditch

Dr David Hilditch (Centra at the University of Lisbon) will be visiting the School of Mathematical Sciences, Queen Mary University of London from the 8 to 19 January 2018. He will give a talk on his current research on critical collapse of solutions to the Einstein field equations on 10 January at 16:30 as part of the college's Relativity & Cosmology seminar series. Dr Hilditch works at the interface between numerical and mathematical relativity with a particular interest on formulations of the Einstein field equations which are suitable for both analytical and numerical analysis. He was until recently based at the Institute of Theoretical Physics of the University of Jena, Germany.

For information regarding Dr Hilditch's visit contact Juan A. Valiente Kroon at the School of Mathematical Sciences, Queen Mary University of London (j.a.valiente-kroon@qmul.ac.uk). The visit is supported by an LMS Scheme 4 (research in pairs) grant.

Perspectives on the Riemann Hypothesis

4 – 7 June 2018
School of Mathematics, University of Bristol

A meeting on the Riemann Hypothesis, and on the theory of the zeta-function and other L-functions.

Speakers to include:

E. Bombieri* (IAS, Princeton)
A. Booker (Bristol)
A. Connes (IHES)
A. Florea (Stanford)
H. Iwaniec (Rutgers)
N. Katz (Princeton)
K. Matomäki (Turku)
P. Michel (EPFL)
S. Patterson (Göttingen)
M. Radziwill (McGill)
C. Skinner (Princeton)
K. Soundararajan (Stanford)
W. Zhang (Columbia)

*TBC
Organisers: J.B. Conrey (AIM/Bristol), J.P. Keating (Bristol), P. Sarnak (Princeton), A. Wiles (Oxford)

To register your interest, complete the form at **heilbronn.ac.uk/events**

Limited support may be available for PhD students and early career researchers. We also welcome applications for caring costs**.

Please apply using the support section in the registration of interest form.

This event is organised in partnership with: American Institute of Mathematics, University of Bristol, Clay Mathematics Institute, EPSRC, Heilbronn Instititute for Mathematical Research and the National Science Foundation.

American Institute of Mathematics | University of BRISTOL School of Mathematics | CMI | EPSRC Engineering and Physical Sciences Research Council | Heilbronn Institute for Mathematical Research | NSF

**Applies to expenses incurred exceptionally as a result of attending the meeting.

LMS Conference Grants

The Society is pleased to report that in 2016–17, awards totalling £243,742 were made in the support of mathematics conferences.

£192,880 was awarded to the organisers of mathematical conferences held in the United Kingdom, and were used to cover the expenses of principal speakers, and to provide support for UK-based research students and for participants from Scheme 5 countries. A total of £13,050 was awarded to support meetings to celebrate new appointments in mathematical departments. Postgraduate Research Conferences were awarded a total of £37,742 to support speakers and participants at conferences organised and run by postgraduate students.

The list can be found on the LMS website at lms.ac.uk/grants/society-conference-grants.

Applicants wishing to apply for funding for a conference will find further details on the Society's website at lms.ac.uk/grants/research-grants.

Annual LMS Subscription 2017-18

Annual subscription payments, including for publications, for November 2017 – October 2018 become due on 1 November 2017 and should be paid no later than 1 December 2017. If you did not receive the subscription renewals reminder sent in September to all members, please email membership@lms.ac.uk.

Members can view and pay their subscriptions at: lms.ac.uk/user. Subscription rates for 2017–18 and a subscription form can be found at: lms.ac.uk/membership/paying-your-subscription.

If you would like to set up a direct debit to the Society (this requires a UK bank account), please do so with GoCardless.com via your online membership record: lms.ac.uk/user. You can also pay by cheque and credit or debit card. Card payments are accepted online only and can be made via your online membership record: lms.ac.uk/user.

Elizabeth Fisher
Membership Engagement Officer

REPORTS OF THE LMS

Report: LMS Prospects in Mathematics Meeting 2017

Undergrads network during a coffee break

The 2017 LMS Prospects in Mathematics Meeting took place on 7 and 8 of September at the University of Reading. It received very positive feedback from both students and speakers. For example:

"*(It was) the first proper mathematical conference I had ever attended, and the first real opportunity I had to interface with experts from such a varied range of research areas. I was stunned at the quality of speakers in the line-up: professors from Oxford, Cambridge, Edinburgh, UCL, Imperial, Cardiff, and plenty of others were in attendance to give a wide variety of talks on areas ranging from random matrix theory and L-functions to propositional logic, Galois representations and the mathematics of Planet Earth.*" Damien Clarke, student from Aberystwyth.

The talks were were perfectly pitched for an audience of undergraduate mathematicians with varied backgrounds and interests. The speakers were Ginestra Bianconi, Thomas Forster, Ben Green, Jon Keating, Valerio Lucarini, Marco Marletta, Beatrice Pelloni, Sarah Rees, Jennifer Ryan, Samir Siksek, Richard Thomas and Amanda Turner.

Over dinner, there were short talks from mathematicians in industry: Rachel Nash (Williams Formula 1), Dr Paul Childs (Emerson-Roxar) and Dr John Lees-Miller (co-founder of Overleaf).

The event received LMS funding for travel and accommodation for attendees, and for the conference dinner. Further sponsorship came from the Heilbronn Institute for Mathematical Research, the Institute of Mathematics and its Applications, the University of Reading Endowment Fund, and Overleaf.

Rachel Newton
University of Reading

Report: LMS Research Summer School 2017

Kerstin Jordaan, Walter Van Assche and Nalini Joshi

The School of Mathematics, Statistics & Actuarial Science (SMSAS), University of Kent hosted the LMS Research Summer School on *Orthogonal Polynomials and Special Functions* from 26 to 30 June 2017.

The Research School, organised by Peter Clarkson and Ana Loureiro (University of Kent), included three lecture courses given by Kerstin Jordaan (University of South Africa) on *Properties of Orthogonal Polynomials*, Nalini Joshi (University of Sydney) on *Discrete Painlevé Equations* and Walter Van Assche (KU Leaven) on *Multiple Orthogonal Polynomials*, as well as three guest lectures by Andy Hone (University of Kent), Andrei Martínez Finkelshtein (University of Almería, Spain) and Adri Olde Daalhuis (University of Edinburgh). An additional activity was the Aurora@Kent Event Making Waves in which Nalini Joshi spoke about her experiences as a female mathematician working in academia.

There were 43 participants from 19 countries, of whom 33 were postgraduate students.

More information can be found: tinyurl.com/opsf-ss.

Peter Clarkson and Ana Loureiro
SMSAS, University of Kent

Report: LMS Midlands Regional Meeting

A walk in Bradgate Park

The 2017 LMS Midlands Regional Meeting took place at Loughborough University on Monday 18 September, followed by a workshop on *Modern Geometry and Physics* (19-21 September).

The three speakers at the Regional Meeting presented lectures on various aspects of geometry and mathematical physics, the theme of the workshop.

The first speaker, Professor Giovanni Felder (ETH Zurich) started from the set of pairs of commuting matrices and arrived at a beautiful conjectural generalisation of the Macdonald constant term identities. He gave a historical overview with an explanation of the recent progress, which surprisingly involves homological techniques.

The lecture of Professor Nigel Hitchin (Oxford) revealed one more remarkable geometric side of the celebrated Hitchin integrable system, which the speaker called Higgs bundle integrable system. Starting with a nice introduction to the geometric theory of integrable systems as torus fibrations, Nigel used his example to explain the important role of the corresponding critical locus in the topology of the moduli spaces of Higgs bundles.

The third lecture by Professor Nikita Nekrasov (Simons Center for Geometry and Physics, Stony Brook) was a fascinating journey from the classical theory of partitions, going back to the work of the sixteenth LMS President Percy A. MacMahon, to modern theoretical physics. Remarkably, this also revealed the deep links with the first two lectures, which was another demonstration of the conceptual power of the ideas and constructions coming to geometry from theoretical physics.

Hamid Ahmadinezhad and Sasha Veselov

Records of Proceedings at LMS meetings
Ordinary Meeting, 18 September 2017

The meeting was held at the Schofield Building, University of Loughborough, as part of the Midlands Regional Meeting & Workshop on *Modern Geometry and Physics*. Over 40 members and guests were present for all or part of the meeting.

The meeting began at 1.40 pm with the Vice-President, Professor John Greenlees, in the Chair.

There were no members elected to Membership at this Society Meeting.

Six members signed the Member's Book and were admitted to the Society.

Professor Alexander Veselov, Professor of Mathematics at the University of Loughborough, introduced the first lecture given by Professor Giovanni Felder (ETH Zurich) on *Pairs of commuting matrices and Macdonald identities*.

After tea, Professor Veselov introduced the second lecture by Professor Nigel Hitchin (Oxford) on *The critical locus of the Higgs bundle integrable system*.

Professor Veselov then introduced the final lecture given by Professor Nikita Nekrasov (Simons Center, Stony Brook) on *Magnificent Four*.

Professor Greenlees thanked the speakers for their excellent lectures. Professor Greenlees then expressed the thanks of the Society to the organisers, Professor Alexander Veselov and Dr Hamid Ahmadinezhad, both of the University of Loughborough, for a wonderful meeting and workshop.

Afterwards, a wine reception and dinner were held, the latter at the Tarboush Restaurant in Loughborough.

Records of Proceedings at LMS meetings
General Meeting, 30 June 2017

The meeting was held at BMA House, Tavistock Square, London. Over 60 members and visitors were present for all or part of the meeting.

The meeting began at 3.30 pm with the President, Professor Simon Tavaré, FRS, in the Chair.

On a recommendation from Council it was agreed to elect Professor Chris Lance and Professor Rodney Sharp as scrutineers in the forthcoming Council elections. The President invited members to vote, by a show of hands, to ratify Council's recommendation. The recommendation was ratified unanimously.

Nine people were elected to Ordinary Membership: Dr Andrea Cangiani, Miss Ola Hajj Sleiman, Dr Rachel Hilliam, Dr Adam Johansen, Mr Johnny Kelsey, Dr Zoltan Leka, Professor Dr Dominik Michels, Dr Erin Pichanick and Dr G. Suseendran.

Five people were elected to Associate Membership: Mr Yasin Elmaci, Mr Oliver Frolovs, Dr Zhangyi He, Mr James Macpherson and Mr David Sheard.

Two members signed the book and were admitted to the Society.

The President, on Council's behalf, proposed that following people be elected to Honorary Membership of the Society: Professor Persi Diaconis, of Stanford University and Professor Étienne Ghys, of the École Normale Supérieure de Lyon. This was approved by acclaim. The President read a short version of the citations, to be published in full in the *Bulletin of the London Mathematical Society*.

The President then announced the awards of the prizes for 2017:
Pólya Prize: Professor Alex Wilkie FRS (University of Oxford)
Senior Whitehead Prize: Professor Peter Cameron (University of St Andrews)
Senior Anne Bennett Prize: Professor Alison Etheridge (University of Oxford)
Naylor Prize & Lectureship: Professor John Robert King (University of Nottingham)
Berwick Prize: Kevin Costello, Perimeter Institute, Canada
Whitehead Prizes: Professor Julia Gog (University of Cambridge); Dr András Máthé (University of Warwick); Dr Ashley Montanaro (University of Bristol); Dr Oscar Randal-Williams (University of Cambridge); Dr Jack Thorne (University of Cambridge); Dr Michael Wemyss (University of Glasgow)

The President introduced a lecture given by Professor Sheehan Olver (Imperial College/University of Sydney) on *The Sokhotski–Plemelj theorem, singular integrals, and solving PDEs*.

Following a break for tea, the President introduced the second lecture by Professor Gwyneth Stallard (Open University) on *Complex analysis and complex dynamics: fruitful interactions*.

At the end of the meeting, the President thanked both speakers for their brilliant lectures.

The President also thanked Darren Crowdy (Imperial College London) and Rob Halburd (UCL) for organising the Graduate Student Meeting in the morning and Dan Nicks (University of Nottingham) and Jean-Marc van den Broecke (Imperial College London), who gave talks at the Graduate Student Meeting. David Kohan Marzagão (King's College London) was also congratulated on winning the prize for the best Graduate Student talk.

After the meeting, a reception was held at De Morgan House, followed by a dinner at the Blue Door Bistro in the Montague on the Gardens Hotel.

Roger Penrose in Warsaw: a Speaker and His Audience

PAUL TOD

Prompted to wonder by the strength and warmth of the response to a public lecture he gave last December, I present some history of Roger and his remarkable family and career, and some memories of my own experience as his graduate student in the 1970s.

December in Warsaw

In December of 2016 it was my good fortune to be in Warsaw for two scientific meetings, a two-day workshop on Roger Penrose's Conformal Cyclic Cosmology followed at once by a two-day meeting on Gravitational Waves. Roger was speaking at both and in the evening between the two meetings he also gave a public lecture on his most recent book *Fashion, Faith and Fantasy*. The venue was the largest lecture theatre in the Physics Department of Warsaw University, which expands to seat about 400, and there was a video-link to a sizable overflow room. The main room was completely full and I was among more than a hundred in the overflow room. I had seen the talk online[1] and when Roger had covered 'fashion' and 'faith' and already an hour had elapsed I wondered what he would do, but he soldiered straight on through 'fantasy'. The audience was fully attentive and after the ninety-minute lecture a large section of it descended on the lectern for more questions, a certain amount of gift-giving, and a great deal of book-signing and selfie-taking. Roger got away something like two and a half hours after starting to talk, holding gifts that he'd been given.

It was a remarkable performance and demonstration, and I was left wondering: where does this degree of commitment, from the audience and from the lecturer, come from? For ninety minutes Roger had talked about extra dimensions in string theory, the measurement problem in quantum mechanics, Conformal Cyclic Cosmology and inflation, all complicated questions, and the audience had listened. There was no condescension from the lecturer, and the audience knew they were getting the real thing, a serious attempt to explain and inform, a genuine intellectual adventure.

Roger Penrose lecturing

Biographical background

To try to answer my question above, I'll start with some biographical background.

Roger's paternal grandfather was James Doyle Penrose, described on Wikipedia as 'an Irish painter', who married the Hon. Elizabeth Josephine, the daughter of Alexander, 1st Baron Peckover, a wealthy Quaker banker. They had four sons. Alexander was the oldest; then Roger's father Lionel; next Roland, the surrealist painter, friend of Picasso, founder of the ICA and spouse of the celebrated photographer Lee Miller (known in parts of Oxford therefore as 'Roger's aunty'); and finally Bernard known as 'Beakus', who lived on the fringes of Bloomsbury, was sometime lover of Dora Carrington and the author of several atmospheric short films (posted on Youtube)[2]. The family lived in Peckover House, Wisbech, which now belongs to the National Trust, and interesting family memorabilia can be seen on the NT website, including a facsimile of a jigsaw made by Lionel, reminding the

[1] It can be seen here: www.maths.ox.ac.uk/node/23396

[2] Roger told me that at some stage the brothers were distinguished as 'Gentleman Penrose, Loony Penrose, Painter Penrose and Sailor Penrose'.

viewer of Roger's later works[3]. Lionel went straight from school into the Friends' Ambulance Unit in 1916, and began the Moral Sciences Tripos in 1919. He met Freud while pursuing postgraduate study in Vienna in 1921-22 and was drawn to psychoanalysis. At that time Freud thought psychoanalysts should first be medical doctors and Lionel returned to England to train as a doctor. After qualifying as a psychiatrist, he became interested in hereditary mental illness and built a career there instead, rising to be Galton Professor of Human Genetics at UCL (and elected FRS in 1953). There is a charming memoir of Lionel online which, among other splendid stories, describes his strong attachment to his Down syndrome patients[4].

Roger's maternal grandfather was John Beresford Leathes, a physiologist and biochemist. He was elected FRS in 1911, making Roger and his brother Oliver third-generation Fellows of the Royal Society. J. B. Leathes married Sara Mara (or Sonia Maria) Natanson, a concert pianist from a Jewish family in Latvia, in 1896 and Lionel married their daughter Margaret in 1928.

Roger was the middle of three sons of Lionel and Margaret, between the mathematician Oliver and the chess master Jonathan, with a younger sister Shirley who is Professor of Cancer Genetics at St George's UL. Roger gives a glimpse of his childhood life in [25], walking in the Suffolk countryside with his brothers and his father, all spread out along the path with Oliver and Jonathan, at the front and rear, playing blindfold Kriegspiel, Lionel as umpire in the middle, and Roger carrying the moves between them. In this memoir of his father [25] Roger acknowledges his great influence and particular pleasure in explaining scientific and mathematical matters. The evening before Roger was about to begin calculus at school, Lionel took him aside "and told me *himself* the essentials of the calculus . . . he was determined to have the pleasure of doing this himself". An illustration of this influence is the development of 'impossible objects' (see "Impossible objects").

In the Biographical Memoir [1] one reads of Lionel ". . . his Quaker upbringing no doubt played an important part in determining his extreme dislike of show and pretentiousness" and these are qualities one recognises in Roger. Coming from a quite remarkable family, he has the sharpest of intellects, a true humility, and a mission to explain, and he is not intimidated by reputation.

[3]See www.nationaltrustcollections.org.uk/object/781539.2

[4]See www.genetics.org/content/150/4/1333

Impossible objects

The story of Roger's involvement with impossible objects is told in [20]. Roger attended the ICM in Amsterdam in 1954 as a research student and visited an associated exhibition of the works of M. C. Escher. Fascinated by what he saw, he was motivated to seek to depict impossible objects and devised the now familiar impossible triangle or tribar. When he next met his father, he showed him the impossible triangle and Lionel at once sought more figures in a similar vein, devising among other things the impossible staircase. These led to the article in the British Journal of Psychology [8], which acknowledged the inspiration from Escher. The Penroses then sent a reprint of this article to Escher who, in the fullness of time, produced his lithographs *Ascending and descending* and *Waterfall* based on the impossible staircase and tribar, in turn acknowledging the Penroses' paper.

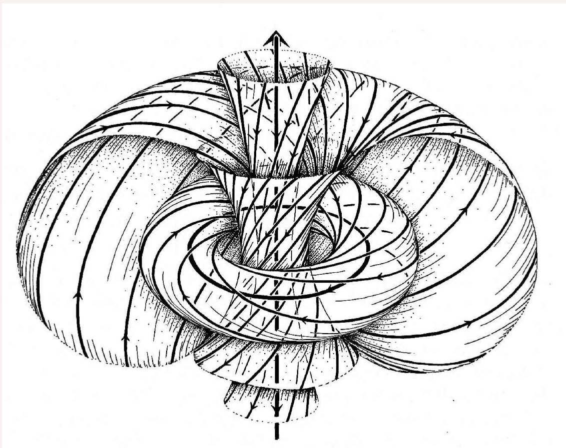
It was subsequently noted that impossible figures had been devised and explored by the Swedish artist Oscar Reutersvärd from 1934 onward and indeed had had an earlier life, as actual or exemplary errors in perspective, as for example in Piranesi's *Carceri XIV* and Hogarth's *Frontispiece to Kirby*. What none of these authors had noticed, unsurprisingly, was the connection to cohomology which Roger pointed out in [23] and [22]. The idea can be described via the tribar: one may cover the figure as drawn by three neighbourhoods, one around each apparent vertex, in each of which it correctly represents an object in space, while globally it does not, which is the essence of cohomology.

Oxford

I want to jump now to 1973. I was a second-year graduate student of Dennis Sciama in Oxford, having followed him from Cambridge, where in the Department of Applied Mathematics and Theoretical Physics his recent students had included Stephen Hawking, Brandon Carter and Martin Rees. I was considerably

Twistors

Roger regularly credits his appreciation of the *Robinson congruence*, a particular congruence of shear-free null geodesics in Minkowski space $\mathbb{M}$ studied by Ivor Robinson, with playing a critical role in the origin of twistor theory [11]. The congruence is illustrated in Roger's sketch. The nested tori and the circles lying on them are obtained from Clifford parallels in $\mathbb{S}^3$ by stereographic projection into $\mathbb{R}^3$, thought of as a constant time hypersurface S in $\mathbb{M}$. The tori collapse to a central degenerate circle. Future pointing null vectors are obtained from the sum of the unit (forward) tangents to the circles and the unit (future) time-like normal to S and these vectors generate the Robinson congruence. There are six real dimensions of Robinson congruences: three for the centre of the central circle, two for the plane it lies in and one for its radius, counted positive or negative according to the sense of the twist. If the circle degenerates to a point then the congruence degenerates to the set of null geodesics incident with a particular null geodesic.



A Robinson congruence (Image by Roger Penrose.)

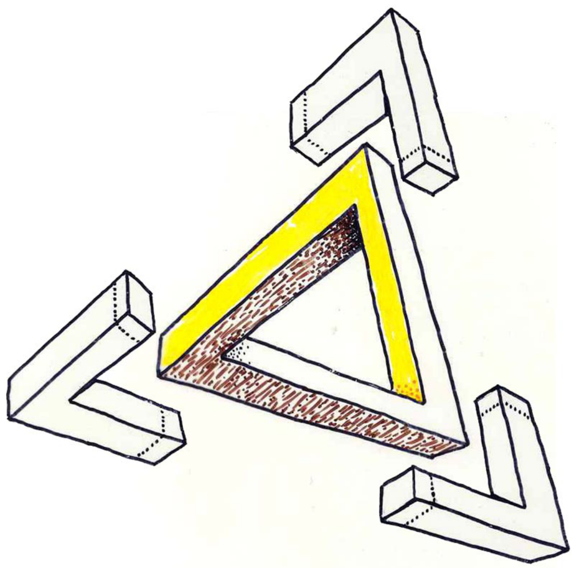
Since the set of (unscaled) null geodesics in $\mathbb{M}$ is five-dimensional, the set of Robinson conguences has a chance to form a complex manifold with the null geodesics as a real hypersurface, and indeed this happens: including various points at infinity, the Robinson congruences correspond to the points of $\mathbb{CP}^3$, now called projective twistor space, $\mathbb{PT}$, and then the null geodesics of $\mathbb{M}$ correspond to the points of a real hypersurface $\mathbb{PN}$. This turns out to be the zero set of a $(2,2)$-signature pseudo-Hermitian form Σ that divides $\mathbb{PT}$ into an upper and lower half, $\mathbb{PT}^+$ and $\mathbb{PT}^-$ respectively. The linear transformations of $\mathbb{PT}$ preserving Σ and the holomorphic volume-form form the group $SU(2,2)$ which is a finite cover of the conformal group $C(1,3)$ of compactified Minkowski space $\mathbb{M}_c$, and this leads to a conformally-invariant calculus. There is a correspondence between points of $\mathbb{M}$ and complex projective lines in $\mathbb{PT}$ which lie in $\mathbb{PN}$: a point p is identified with the set of real null geodesics through it; this is the celestial sphere of p and is naturally a $\mathbb{CP}^1$, $\mathbb{L}_p$ say. Complex projective lines not in $\mathbb{PN}$ then define points of the complexification $\mathbb{CM}_c$ of $\mathbb{M}_c$. This can be understood in terms of the Klein quadric, $\mathbb{Q}$: the lines in $\mathbb{CP}^3$ lie on a quadric $\mathbb{Q}$ in $\mathbb{CP}^5$, the points of which are interpreted as (complex) null rays lying on the complexified $O(2,4)$ null cone, which is one way of defining $\mathbb{CM}_c$.

In 1968 Roger published [12] a construction of solutions of the zero rest-mass free-field equations in regions of $\mathbb{M}$ from functions on regions in $\mathbb{PT}$: let U be a neighbourhood of $p \in \mathbb{M}$ that corresponds to a neighbourhood V the line $\mathbb{L}_p$ and suppose $f(Z)$ is a holomorphic function of the twistor variable Z on V, then restrict f to $\mathbb{L}_q$ for $q \in U$ and, provided f has suitable singularities, perform a contour integral around them to obtain a space-time field $\phi(q)$ which satisfies the zero rest-mass free-field equations. The proper understanding of these integrals and the proper definition of the functions f as elements of first sheaf cohomology groups of V was clarified by Roger in [17], after he was introduced to sheaf theory, which in turn led to [3]. This can be thought of as the culmination of 'flat' twistor theory. At much the same time Roger following Ted Newman gave a construction for the general Ricci-flat, anti-self-dual 4-metric by holomorphic deformation of the neighbourhood of a complex projective line in $\mathbb{PT}$, and Richard Ward gave a construction for the general anti-self-dual Yang–Mills field in terms of holomorphic vector bundles over the neighbourhood of a complex projective line in $\mathbb{PT}$. These were the first results in what can be called 'curved' twistor theory.

daunted, and not really at home in the Department of Astrophysics, so was very pleased to be able to move to the Mathematical Institute and Roger's supervision when he came from Birkbeck to be Rouse Ball Professor.

As a graduate student one may not be so well-informed about one's supervisor but, in retrospect, Roger was clearly the strongest relativist in the UK at that time, and probably the strongest in the world. In the previous ten or fifteen years he had transformed the subject by introducing geometrical insights and techniques which now are completely central. Highlights would be the use of two-component spinors from 1960, which transformed calculations in coordinates; the geometrisation of asymptotics by adding boundaries at infinity from 1962; Penrose diagrams and the notions of causal space theory, leading to [13]; and the first singularity theorem, from 1965. The last was enormously significant as it was still possible at that time to regard space-time singularities as an artefact of the high symmetry of known examples. Roger's discovery of the trapped surface was the key to showing, in a coordinate-independent way and without the need for symmetry, that (with other conditions of course) a space-time singularity was inevitable transformed expectations. From now on one expected gravitational collapse both to occur and to lead to singularities. His work with Hawking seeking the most general singularity theorem led to their result of 1970 and a body of work which was rewarded with the 1988 Wolf Prize. I should also mention Roger's Cosmic Censorship Hypothesis of 1969, according to which if space-time singularities form in an evolution from regular data, then generically they are always concealed behind a horizon[5]. This has been proved in some cases with symmetry and is still guiding research. Roger's unrivalled geometrical grasp of general relativity enabled him to discover results like those in [9] or [16] which it's likely nobody else would ever have seen.

When he arrived in Oxford, Roger's interest was largely centred on twistor theory (see "Twistors"). Its origins went back further, but the first publication was in 1967 [11] and what became for us the bible of the subject 'Penrose and MacCallum' [14] had appeared the previous year. According to Penrose and MacCallum, twistor theory was 'an approach to the quantisation of fields and space-time', the idea being that a new formalism would indicate new directions for research, and more specifically that the nonlocal nature of the relation between space-time and twistor space — a point of space-time becomes a complex projective line in twistor space, while a point in twistor space becomes an extended object, in some cases a null geodesic, in space-time — would lead to a picture of the quantisation of space-time as smearing out or blurring points rather than (say) blurring the metric and therefore the light-cones[6]. Crucial to the development of twistor theory was the presence in Oxford as Royal Society Research Professor of Michael Atiyah. He, with Nigel Hitchin and later Mike Eastwood, introduced Roger and the group to sheaf cohomology, the theory of several complex variables, and the deformation theory of complex manifolds of Kodaira and others, which turned out to be crucial.



A tribar showing the connection to cohomology (Image from Penrose's hand-drawn slides.)

Roger brought with him from Birkbeck his practice of Friday meetings, which ran from noon until exhaustion set in and whose form may have owed something to Quaker meetings. There would be a first phase of *questions*, on lectures or seminars heard or papers read, and a second phase of *answers*, in the sense of new results from the group. From 1976, these answers found a more permanent place in the Twistor Newsletter, hand-written in those pre-LaTeX days, but still available at [29]. It was in a Friday meeting that, when Roger was describing the twistor contour integral solution of the zero-rest-mass equations, somebody said 'That looks like a sheaf',

[5] There exist more precise formulations; see e.g. [28].
[6] See figure 33.7 in *The Road to Reality*.

Aperiodic tiles

Penrose tilings are ubiquitous. They can be walked on in Oxford, Austin, San Francisco, Helsinki and doubtless elsewhere, and at one time could be bought as a jigsaw puzzle. Roger's interest in tilings goes back to his childhood when he recalls his father telling him that the earth could not be tiled by hexagons. In fact Lionel owned a copy of Kepler's 'Harmonices Mundi' in which one of the figures would seem to show Kepler trying to tile the plane with decagons and other shapes similar to those in [15] (just google 'Kepler's Aa').

An aperiodic set of proto-tiles is one which will tile the whole plane, given sufficient quantities, but *only* in a nonperiodic way. The first such known was a set of 20,426 proto-tiles published by Berger in 1964 in response to a challenge from his supervisor the logician Wang. There was subsequently some interest in reducing the number needed. Roger published a set of six in [15] based on decagonal symmetry, which is forbidden as a crystal symmetry. After a lecture he gave on these, he was told that Raphael Robinson had also produced a (completely different) set of six. Roger set about reducing the number of proto-tiles, getting it down to just two in two different ways: the kites and darts, which gave rise to the chickens and ducks jigsaw puzzle, and the fat and thin rhombi [19]. These tilings were made known to a much wider audience by Martin Gardner's article [4] in 1977.

The next part of the story is the discovery of quasicrystals by Shechtman in 1982, published 1984. These are crystals of metal alloys which show in electron diffraction patterns the forbidden ten-fold symmetries, and Shechtman came in for some intemperate criticism (e.g. 'There are no quasicrystals — only quasi-scientists') though he was eventually recognised by the award of the Nobel prize in Chemistry in 2011. Almost simultaneously with Shechtman's publication, Levine and Steinhardt [6] introduced the term 'quasicrystal' for ordered structures derived from Roger's tilings, seeking to give a theoretical understanding of Shechtman's observations.

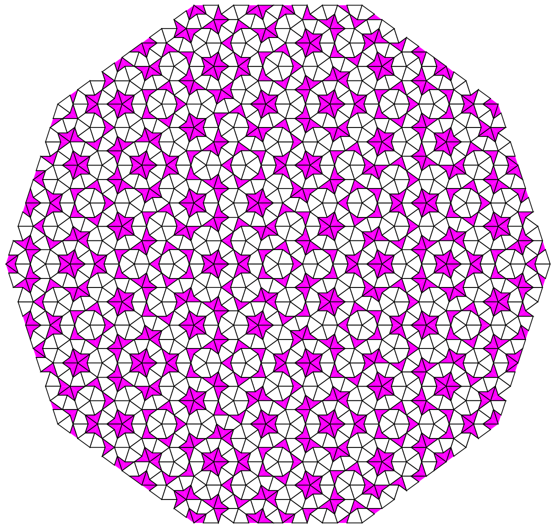
Two rather Penrosian themes come together in this area. On the one hand it was shown by Raphael Robinson [27], following earlier work of Berger, that the tiling problem (given a set of proto-tiles will it tile the plane?) is undecidable. On the other, a Penrose tiling is not determined by local rules of assembly: there is an infinite sequence of constraints which must be observed as the tiling is assembled in order for it to cover the plane. This raises the question, posed in [21], of how a physical quasicrystal forms — this is presumably a quantum mechanical process of adding atoms or molecules to a growing crystal, but how can these nonlocal rules then be enforced?

Another anecdote: when Roger moved into a new house near Oxford, he was surprised to discover in one of the bathrooms a toilet roll printed with a version of one of his non-periodic tilings, possibly left as a tease by the previous owner. I've always been amused by this story as nothing is more quintessentially periodic than a toilet roll.

A nonperiodic paving outside the Mathematical Institute, Oxford an observation made concrete by Roger in Twistor Newsletter 2 of 1976, and culminating first in [3] and later as the 'Penrose Transform' in the book [2][7]. It was also in a Friday meeting that Roger announced the non-linear graviton — this is a construction to obtain the general, anti-self-dual, Ricci-flat metric from a holomorphic deformation of a region of twistor space. It was inspired by Newman's $\mathscr{H}$-space construction [7], needs Kodaira theory and can claim to be the most significant success of so-called 'curved twistor theory'. This event can be dated fairly accurately as it was in the period of creative liberation experienced (by some of us) following the resignation of Richard Nixon in August 1974. George Sparling, who had accompanied Roger from Birkbeck, produced the first explicit examples of the construction almost at once. A little later, Richard Ward gave a construction for the general anti-self-dual Yang–Mills field in a region of flat space in terms of a holomorphic vector bundle on the corresponding region of twistor space. These two 'curved' twistor constructions generated great interest among mathematicians, since anti-self-dual fields can be real in Riemannian signature which mathematicians typically favour, while they remain complex in Lorentzian signature. Roger wanted twistor theory to be about the world, which he firmly believes to be four-dimensional and Lorentzian, but from about 1978 it was the Riemannian side of twistor theory which grew faster.

The atmosphere of the Friday meetings was geometrical in content and playful in manner. The board was typically covered in Roger's 'bug' or graphical notation for tensor calculus (see Wikipedia's entry on Penrose graphical notation[8], which had so amused Roger's supervisor John Todd when he saw it in the 1950s), or tables of commutators for moves on Rubik's cube when that passed through like a tornado in 1980. From about 1974 when Penrose tilings were evolving (see "Aperiodic tiles") examples of the different generations of the tilings were scattered about the room, with other mathematical models, teetering piles of papers with labels like 'Unanswered correspondence 1973-75' and a tribar scaled-up in dimension — a 3-dimensional model of an object impossible in four dimensions which was intended to provoke curiosity and mirth in any passing 4-dimensional beings.

Throughout the early 80s there was an emphasis on the material which became the two volume *Spinors and space-time*. Once these two volumes, optimistically described as 'forthcoming' by Pirani in 1964, had come forth in 1984 and 1986, Roger was more occupied with the big unanswered questions that informed his popular books.



Kites and darts

Popular books

I've called these the 'popular' books, and they have been popular, but for the reader they are a different prospect from the familiar genre of popular science which one can trace through many British cosmologists — Eddington, Hoyle, Sciama and Paul Davies, among others. Hawking famously recalled being told by his editor that every formula included would halve the number of readers but Roger's *The Road to Reality* has formulae and diagrams on more than half of its thousand pages. Roger has always used these books to express serious scientific theses, developed at length, that are difficult to place elsewhere in the scientific literature. An early sign of this aspect of his work is his article [18] for the Einstein Centennial volume. In this Roger introduced the Weyl Curvature hypothesis (see "Conformal cyclic cosmology") and developed his criticisms of quantum mechanics and the puzzle of collapsing wave functions, along with his thoughts on time-asymmetric physics — a whole set of original ideas which one might not expect to see in a centennial survey.

In *The Emperor's New Mind: Concerning Computers, Minds, and The Laws of Physics* (1989) Roger

[7] I once overheard Oliver Penrose at a conference telling somebody 'No, it's my brother who has the transform.'

[8] https://en.wikipedia.org/wiki/Penrose_graphical_notation

Conformal cyclic cosmology (CCC)

Conformal cyclic cosmology (CCC) is Roger's 'outrageous suggestion' [26] combining his Weyl Curvature Hypothesis with the observation that there is a positive cosmological constant in the universe. The Weyl Curvature Hypothesis (WCH) is set out most completely in [18]. Roger claims it is an observational fact that the Big Bang, regarded as a singularity of a Lorentzian manifold, was very special. The argument goes as follows: there is such a thing as the Second Law of Thermodynamics, according to which the entropy of the world increases into the future; there is also a consensus that the matter content of the world was in thermal equilibrium at a very early stage, which equivalently means that the matter was in its highest entropy state; so some other component of the world must have been in a low entropy or *special* state. General Relativity equates the matter content to the Ricci curvature, which is ten components of the Riemann curvature, the other ten being the Weyl curvature, which is often described as the free gravitational field. Thus it must be the Weyl curvature which was special at the Bang, and Roger proposes the simplest hypothesis: that it was *zero* at the Big Bang singularity — in conversation, he allowed the possibility that it was simply small but essentially this is the WCH. I should say that Roger also has a persuasive quantitative argument for this conclusion. Now, intuitively, it is difficult to *cause* something which happens at time zero and in [18] Roger entertained the possibility of time-asymmetric laws of nature, specifically of quantum gravity, acting backwards in time, but CCC provides a better way to enforce the WCH. It is also difficult to characterise singularities of Lorentzian manifolds at which the metric, connection and Ricci curvature are singular but the Weyl curvature is not. A class of examples, simple to describe but in fact quite general given sufficient regularity conditions, is constituted by space-times which can be conformally extended through the Big Bang singularity: if the physical metric is related to an unphysical but nonsingular metric by a conformal factor vanishing at the Bang then the physical Ricci curvature will be singular while the physical Weyl curvature will not. I should add that one wants the Bang surface, when added as a boundary, to be space-like because of a belief in the Cosmic Censorship Hypothesis, another of Roger's suggestions.

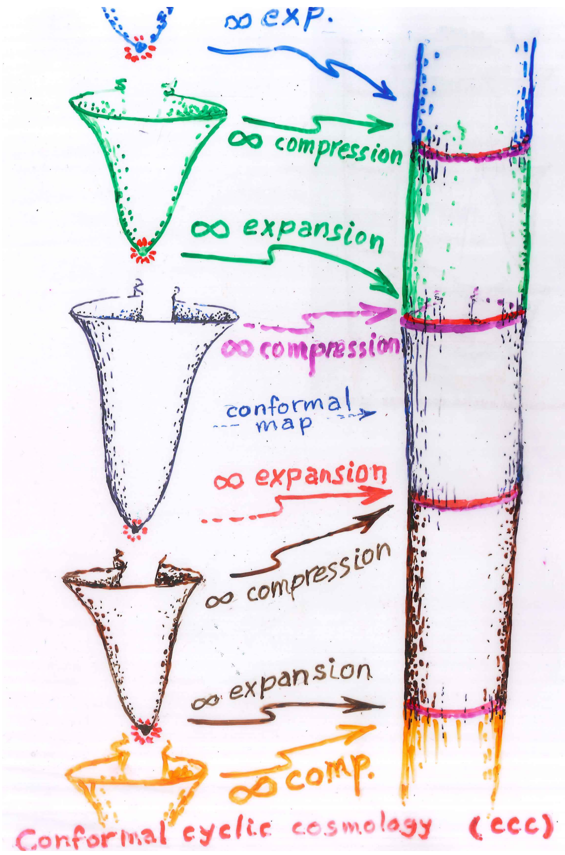
Some years later the existence of a positive cosmological constant was established and those familiar with Roger's 1965 classic [10] would have realised at once that this entailed the existence of a space-like $\mathscr{I}$, pronounced 'scri': that is to say that there would exist a conformal rescaling of the physical metric which permitted the adding of a future conformal boundary ($\mathscr{I}$) which would be space-like. This time the physical metric is related to an unphysical conformal metric that extends through infinity by a conformal factor which becomes infinite. It also follows, another result in [10], that the Weyl curvature necessarily vanishes at $\mathscr{I}$. Now one can see the outrageous suggestion coming: one wants a condition at the Bang to the effect that the Weyl curvature is zero there; with a positive cosmological constant one necessarily has a space-like future conformal boundary at which the Weyl curvature vanishes. Clearly then one should glue these two boundaries together. There will be a conformal metric extending backward through the Bang and forward through $\mathscr{I}$. It will be related to the physical metric by a conformal factor which vanishes at the initial Bang and goes to infinity at the final $\mathscr{I}$ — this will be a whole *aeon* in Roger's terminology — then the conformal factor will be replaced by its inverse and a new aeon will follow.

In this model the conformal metric is *cyclic*, but not necessarily *periodic*, while a sequence of physical metrics runs from the infinitely small to the infinitely large. Any parts of physics which are conformally invariant need not regard the cross-over surface as a barrier. Thus one should foresee that gravitational or electromagnetic radiation will pass through. This raises the possibility of detecting in one aeon effects emanating from the previous aeon and Roger has suggested how one might be able to observe circular disturbances in the cosmic microwave background in our aeon with an origin in the collision of super-massive black holes in the previous aeon. Not surprisingly, this is controversial and claims that these circles have been observed have led to heated debate.

questioned the possibility of 'strong AI' by arguing, from Gödel's theorem, that human consciousness is non-algorithmic. The suggestion that parts of physics could be noncomputable was made a few years earlier (see [5]) but Roger went further here in suggesting that some part of the physics underlying human thought and consciousness must have this property, and that quantum mechanics was the place to look. The suggestion that wave functions objectively collapse and that gravity is involved in the process is also made here, and nonperiodic tilings and quasicrystals are invoked in a discussion of nonlocality.

His next book *Shadows of the Mind: A Search for the Missing Science of Consciousness* (1994) can be seen as a sequel. The earlier book had led to some hostility and ill-informed responses, accusing Roger of misunderstanding pretty much everything, or of privileging mathematicians or even himself as a kind of Übermensch who alone really understood the world. The second book sought to clarify and extend the argument that human consciousness is non-algorithmic, that what was needed to understand consciousness was new science but was still science, and to make a suggestion of what it was. The suggestion is 'that there be large-scale quantum-coherent behaviour ... occurring within the microtubules in the cytoskeletons of neurons' [24]. This time biologists joined philosophers and the AI community in the chorus of rejection[9] but, like many of Roger's suggestions, this is still a live issue.

I've mentioned Roger's next book already. *The Road to Reality*, subtitled *A Complete Guide to the Laws of the Universe*, (2004) reads like a 1000-page attempt to challenge received views and to teach a critical attitude to the laws of physics, still unintimidated by reputation. It's avowedly pedagogic and the closest of Roger's books to teaching calculus to your child (though a good deal more ambitious). It paved the way for *Fashion, Faith, and Fantasy in the New Physics of the Universe* which, although it was published in 2016 is an account of lectures given in 2003 which must have drawn on the experience of writing *The Road to Reality*. Having warned of the potential of *fashion* — specifically for higher dimensional physical theories — and *faith* — here faith in quantum theory when the problem of collapse is still unresolved — to mislead, Roger has some kind words for *fantasy* and the play of imagination, which he exemplifies by his own 'outrageous suggestion' of conformal cyclic cosmology (again see "Conformal cyclic cosmology"). This is in turn the subject of *Cycles of Time* (2010).



Conformal cyclic cosmology (Image from Penrose's hand-drawn slides.)

What has seemed to me a unique and unregarded value in these books is that they contain a record of the introspection and the phenomenology of creative thought of a great intellect — why would anybody not listen to that? And that, I'm sure, is what the audience in Warsaw and a dozen similar venues where Roger has given public lectures, turn up for, and get.

Acknowledgements

I would like to thank the LMS for the opportunity to write this memoir, and acknowledge that it has been greatly facilitated by my involvement in Heidi Morstang's project to make a film about Roger. I'm enormously indebted to Roger for his teaching and personal example in the forty-odd years I've known him.

[9] Two personal favourites among the reviews: 'Mr *[sic]* Penrose ... is persuaded by an argument that all experts in mathematical logic have long rejected as fallacious' and 'Nothing we have said in this paper demonstrates the falsity of the quantum-consciousness connection. Our view is just that it is no better supported than any one of a gazillion caterpillar-with-a-hookah hypotheses.'

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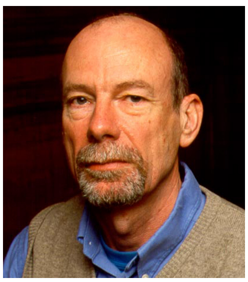

Paul Tod

I have recently retired from 33 years as Fellow and Tutor at St John's College and University Lecturer at the Mathematical Institute in Oxford. The interview for this post was the only successful job interview I've ever had, so, younger colleagues, persevere! You may only have to be lucky once.

Integration in Elementary Terms

DAVID MASSER

We explain this classical notion and describe a fundamental result of Liouville. We explain also why the main interest lies in the integration of algebraic functions. In principle the basic problem here was solved by Risch in 1970, but this does not suffice to verify an assertion of Davenport from 1981 about families of algebraic functions. We describe how recent work of Zannier and the author establishes a corrected version.

Elementary integration

Ever since the invention of the differential and integral calculus (still an emotional topic — in Basle I once saw a French-speaking lecturer in the history of mathematics face a mainly German-speaking audience, and his solution to the problem was to use the "language of the enemy") it has been recognized that integration is much harder than differentiation. Thus

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{\log x}\right) = -\frac{1}{x(\log x)^2}$$

but

$$\int \frac{\mathrm{d}x}{\log x} = ? \tag{1}$$

One can cheat by inventing new names; for example $\int_2^t \mathrm{d}x/\log x$ is more or less $\mathrm{Li}(t)$ used in fine forms of the Prime Number Theorem. Or $\int_0^t \exp(-x^2)\mathrm{d}x$ is more or less the error function $\mathrm{erf}(t)$ in Probability Theory. Come to think of it $\int_1^t \mathrm{d}x/x = \log t$ is also of this type; but that we allow.

We can put a stop to the cheating by demanding that the integral of $f = f(x)$ should involve no essentially new ingredients. We can reasonably expect f itself to turn up, and as for $1/x$ above we must allow "log". On grounds of symmetry we allow also "exp". We will also throw in radicals like $\sqrt{x}, \sqrt[3]{x}, \ldots$ as well as arbitrary algebraic functions such as $\alpha = \alpha(x)$ with $\alpha^5 + \alpha = x$ ("ultraradicals" as in Kasner and Newman [6] pp.17,18). If we can do the integration using combinations of these functions we say (by abuse of adjectives) that f is "elementary integrable" or $\int f(x)\mathrm{d}x$ is "elementary". This is more or less the definition that Liouville started with. We give a formal modern version in the next section.

We will see that the problem of deciding whether a given f is elementary integrable presents the most difficulty (and interest) just for algebraic functions. Actually this led to the discovery of elliptic and abelian functions and all that followed. In such a context we have

$$\int \frac{\mathrm{d}x}{x^2+1} = \tan^{-1} x = -\frac{i}{2}\log\left(\frac{1+ix}{1-ix}\right) \tag{2}$$

where the first answer is provided by Maple; and similarly $\int (x+1)\mathrm{d}x/\sqrt{x^2+1}$ is

$$\sqrt{x^2+1}+\sinh^{-1} x = \sqrt{x^2+1}+\log(x+\sqrt{x^2+1}). \tag{3}$$

So these are elementary.

But already $\int \mathrm{d}x/\sqrt{x^4+1}$ and $\int \sqrt{x^4+1}\mathrm{d}x$ have something to do with inverse elliptic functions (genus $g = 1$ in some sense to be hinted at later) and in fact are not elementary (otherwise one would have no need to define elliptic functions).

However there are surprises: already Euler [2] in 1780 found something equivalent to

$$\int \frac{\sqrt{x^4+1}\,\mathrm{d}x}{x^4-1} = \frac{1}{4}\sqrt{2}\log g_1 + \frac{i}{4}\sqrt{2}\log g_2 \tag{4}$$

with

$$g_1 = \frac{\sqrt{2}x - \sqrt{x^4+1}}{x^2-1}, \quad g_2 = \frac{i\sqrt{2}x + \sqrt{x^4+1}}{x^2+1}$$

in which the two logarithms cannot be combined into a single one. Note that Maple 18 cannot do this (the reason will become clear below). It can (just about) check by differentiation, provided you help out with things like $\sqrt{x^2}$.

It may have been examples like these that prompted Hardy [3] (p.11) in 1905 to write

"... no general method has been devised by which we can always tell, after a finite series of operations, whether any given integral is really elementary, or elliptic, or belongs to a higher order of transcendents."

And over a century later nothing much has changed, even for algebraic functions (for which the above genus is a more precise measure of geometric complexity) although for these the connexion with torsion on abelian varieties (see later) is now much better understood, and algorithms for this torsion have been developed. In particular Risch [9] gave an elegant formulation and sketched a method which should decide if a given algebraic f is elementary integrable. But as yet there is no MSC2010 number for the subject.

We will soon sketch a proof that

$$\int \frac{\mathrm{d}x}{\sqrt{x^3 - x}} \tag{5}$$

is not elementary. But first we have to define this term properly.

Differential algebra and Liouville

Rosenlicht [10] showed how to avoid analytic problems such as domains of definition (for example with $\log x$ or $\sqrt{x}$ or $\sqrt[3]{\log\log x}$) by working algebraically. A differential field (F, δ) is a field F with a map δ from F to itself satisfying

$$\delta(u + v) = \delta(u) + \delta(v), \quad \delta(uv) = v\delta(u) + u\delta(v).$$

One says that f in F is elementary integrable if there exists a chain

$$F = F_0 \subset F_1 \subset \cdots \subset F_n \tag{6}$$

of differential fields (F_i, δ) with "$\int f$ in F_n", i.e. $f = \delta(g)$ for some g in F_n; further each extension $K'/K = F_i/F_{i-1}$ $(i = 1, \ldots, n)$ has $K' = K(z)$ for one of the following three possibilities:

(A) z is algebraic over K, e.g. "$z = \sqrt{w^3 - w}$" for some w in K, i.e. $z^2 = w^3 - w$,

(L) z is logarithmic over K, "$z = \log w$" for some $w \neq 0$ in K, i.e. $\delta(z) = \delta(w)/w$,

(E) $z \neq 0$ is exponential over K, "$z = \exp w$" for some w in K, i.e. $\delta(z)/z = \delta(w)$.

Thus logarithm and exponential are expressed just with δ.

Incidentally the powerful techniques of modern differential Galois theory are incapable of deciding whether a given algebraic f is elementary integrable.

So to do (1) we could take $F = \mathbf{C}(x, y)$ with $y = \log x$ in the above sense and $\delta = \mathrm{d}/\mathrm{d}x$ so that $\delta(x) = 1, \delta(y) = 1/x$. Here the fact that F has transcendence degree 2 over $\mathbf{C}$ makes the problem relatively easy: we can treat x, y as independent variables. For (5) now $y = \sqrt{x^3 - x}$ with $\delta(x) = 1, \delta(y) = (3x^2 - 1)/(2y)$ coming from

$$y^2 = x^3 - x \tag{7}$$

and we have transcendence degree only 1.

But how to deal with the chain (6)?

Suppose for (5) that (E) came in, e.g. $\int \mathrm{d}x/y = \exp w$ for w in F. Then $1/y = (\exp w)\mathrm{d}w/\mathrm{d}x$ so $\exp w$ is also in F which seems unlikely; "exp doesn't go away on differentiating".

And (L) cannot come in too elaborately, e.g. $\int \mathrm{d}x/y = (\log w)^2$ implies similarly $\log w$ in F. So does $\int \mathrm{d}x/y = \log\log w$. But of course $(\log w)^1$ can come in, e.g. $\int \mathrm{d}x/x = \log x$.

In a series of papers around 1835 Liouville made this precise (without using differential algebra). Any (F, δ) as above has a constant field C defined as the set of c in F with $\delta(c) = 0$.

Theorem (Liouville). *Suppose that C is algebraically closed (in zero characteristic). Then f in F is elementary integrable if and only if there are $f_0, g_1 \neq 0, \ldots, g_m \neq 0$ in F and $c_1, \ldots, c_m$ in C with*

$$\text{“}\int f \;=\; f_0 + c_1 \log g_1 + \cdots + c_m \log g_m\text{”} \tag{8}$$

i.e.

$$f \;=\; \delta(f_0) + c_1\frac{\delta(g_1)}{g_1} + \cdots + c_m\frac{\delta(g_m)}{g_m}.$$

Thus indeed only (L) is allowed, and that in very direct form, so that the chain has greatly simplified. And (3) and (4) are typical examples of (8). See [10] for a modern proof, and also Lützen [7] for a historical account.

Examples

Thus for (5) with (7) we must have $1/y = \mathrm{d}f_0/\mathrm{d}x + \sum_{i=1}^{m} c_i(\mathrm{d}g_i/\mathrm{d}x)/g_i$ or finally in terms of differentials

$$\frac{\mathrm{d}x}{y} \;=\; \mathrm{d}f_0 + c_1\frac{\mathrm{d}g_1}{g_1} + \cdots + c_m\frac{\mathrm{d}g_m}{g_m} \tag{9}$$

for $f_0, g_1, \ldots, g_m$ in $\mathbf{C}(x, y)$ and $c_1, \ldots, c_m$ now in $\mathbf{C}$.

Next the key step is to examine the poles. In $\mathbf{C}(x)$ their meaning is obvious, in $\mathbf{C}(x, y)$ less so; but it looks like

$$y = \sqrt{x^3 - x} = \sqrt{x(x-1)(x+1)}$$

should have zeroes at $x = 0, 1, -1$. In fact one should work on the elliptic curve E_{-1} (this notation will be explained soon) defined by (7), and it suffices to look at the real locus to see the horizontal line $y = 0$ cutting the graph transversally so convince oneself that there are simple zeroes at the points $(0,0), (1,0), (-1,0)$. It is less clear that y has a treble pole at the point O at infinity. Similarly with a vertical line cutting tangentially x has a double zero at $(0,0)$ and a double pole at O. So it may be imagined that $\mathrm{d}x$ has a simple zero at $(0,0)$, and also at $(1,0), (-1,0)$, and a treble pole at O.

Thus $\mathrm{d}x/y$ has no poles, even at O.

Now look at (9). If f_0 has a pole somewhere, then $\mathrm{d}f_0$ has a repeated pole there; e.g. for $f_0 = 1/(x-a)$ we have $\mathrm{d}f_0 = -\mathrm{d}x/(x-a)^2$.

But $\mathrm{d}g/g$ has only simple poles; e.g. for $g = 1/(x-a)^3$ we have

$$\frac{\mathrm{d}g}{g} = -3\frac{\mathrm{d}x}{x-a}. \tag{10}$$

Thus f_0 cannot have any poles. This is impossible (e.g. $x - a$ has a pole at O) unless f_0 is constant. But then $\mathrm{d}f_0 = 0$ in (9).

To deal with the surviving $\mathrm{d}g_i/g_i$ we can take another key step in assuming m minimal. This implies that $c_1, \ldots, c_m$ are linearly independent over $\mathbf{Q}$; e.g. $c_1 = 2c_2$ would give $c_1\mathrm{d}g_1/g_1 + c_2\mathrm{d}g_2/g_2 = c_2\mathrm{d}g/g$ for $g = g_1^2 g_2$, leading to smaller m.

A final key step is to consider residues. Suppose g_i has a pole of order $-k_i$ (maybe $k_i > 0$) at some point P. It turns out that $\mathrm{d}g_i/g_i$ has a pole of order at most 1 with residue k_i (e.g. in (10) the residue is -3). So the residue of $\mathrm{d}x/y$ is $\sum_{i=1}^m c_i k_i$. But this is 0; therefore by linear independence $k_1 = \cdots = k_m = 0$ and so $g_1, \ldots, g_m$ have no poles anywhere. Thus as before $\mathrm{d}g_1 = \cdots = \mathrm{d}g_m = 0$ in (9) too, an absurdity.

The reader is invited to construct a similar proof for (1) exploiting the independence of x and $y = \log x$; but now no elliptic curve is needed. One could also replace x by e^x and work with functions meromorphic on $\mathbf{C}$ (and possibly a dash of Nevanlinna). For more hints see [10].

Torsion on abelian varieties

Despite the grand-sounding title, this section will treat only elliptic curves, which are one-dimensional abelian varieties.

Let us consider

$$\int \frac{\mathrm{d}x}{(x-2)\sqrt{x^3 - x}} \tag{11}$$

instead of (5). Here we do have poles at $P^+ = (2, \sqrt{6})$ and $P^- = (2, -\sqrt{6})$, both simple. The above arguments show that $\mathrm{d}f_0 = 0$ and $g_1, \ldots, g_m$ have no poles except possibly at P^+, P^-. In standard divisor notation we may write $(g_i) = k_i^+ P^+ + k_i^- P^-$. By general curve theory the degree $k_i^+ + k_i^- = 0$ and by elliptic curve theory the point (not divisor)

$$k_i^+ P^+ + k_i^- P^- = k_i^+(P^+ - P^-) = 2k_i^+ P^+ = O.$$

Some $k_i^+ \neq 0$, and it follows that P^+ is torsion.

This is the heart of the connexion between integration of algebraic functions and torsion on abelian varieties. A general formulation is in [9]. For our example torsion is not difficult to disprove and indeed

$$2P^+ = \left(\frac{25}{24}, -\frac{35}{288}\sqrt{6}\right), \ 4P^+ = \left(\frac{1442401}{117600}, ?\right),$$

$$8P^+ = \left(\frac{4386303618090112563849601}{1402260988295659323350400}, ?\right)$$

(and so on) is pretty convincing (if doubling increases the arithmetic complexity how can the sequence become periodic?). Even more convincing is to eliminate the field $\mathbf{Q}(\sqrt{6})$ by replacing x by $x/6$ and y by $y\sqrt{6}/36$ to get $(12, 36)$ on $y^2 = x^3 - 36x$; the double is $(25/4, -35/8)$ so we can apply what is known in the trade as Lutz-Nagell.

However to disprove torsion over an arbitrary number field is a deep matter, which is why Maple 18 cannot integrate (4) over $\mathbf{Q}(i, \sqrt{2})$.

But we can! On replacing x by $\frac{1-i}{\sqrt{2}}(x-i)/(x+i)$ and at the same time $\sqrt{x^4+1}$ by $(2+2i)\sqrt{x^3-x}/(x+i)^2$ (to resolve a singularity - not strictly necessary but it is reassuring to see E_{-1} again) we find that the differential in (4) becomes $-2i\sqrt{2}\omega$ with

$$\omega = \frac{x^3 - x}{x^4 - 6x^2 + 1}\frac{\mathrm{d}x}{\sqrt{x^3 - x}}.$$

However we now have poles at four pairs of points, whose abscissae are the zeroes of the quartic, and it is not so easy to calculate the (g_i); in particular we cannot deduce that any of these points are torsion. In fact they are, of order 4; so we are certainly not going to prove that Euler couldn't have done what he did!

Next we calculate residues $\mathrm{res}_P\omega$. Denoting the poles as $P^\pm, Q^\pm, R^\pm, S^\pm$ where

$$P^+ = (1+\sqrt{2}, 2+\sqrt{2}), \quad Q^+ = (-1+\sqrt{2}, i(2-\sqrt{2})),$$

$$R^+ = (1-\sqrt{2}, 2-\sqrt{2}), \quad S^+ = (-1-\sqrt{2}, i(2+\sqrt{2})),$$

and the minus superscripts come from changing the signs of the ordinates, we find that all the residues have the form $\pm 1/8$ or $\pm i/8$; for example $\mathrm{res}_{P^\pm}\omega = \pm 1/8$.

On the other hand $\mathrm{res}_P(\mathrm{d}g/g) = \mathrm{ord}_P g$ the order of vanishing of g at P (compare (10) above), so all residues are $\mathbf{Z}$-linear combinations of $c_1, \ldots, c_m$ in Liouville's Theorem. And using again the minimality of m we can easily see that conversely $c_1 \ldots, c_m$ are $\mathbf{Q}$-linear combinations of the residues (such considerations play an important role also in the general theory of elementary integration of algebraic functions). Thus $m = 2$ and we have for example $c_1\mathrm{ord}_{P^\pm} g_1 + c_2\mathrm{ord}_{P^\pm} g_2 = \pm 1/8$.

We now perform some jiggery-pokery to guess c_1, c_2, g_1, g_2. A reasonable start is $c_1 = 1/8,\ c_2 = i/8$; then we can read off the orders and we find

$$(g_1) = P^+ - P^- + R^+ - R^-,$$

$$(g_2) = -Q^+ + Q^- - S^+ + S^-.$$

These are certainly of degree zero as required by general curve theory. And by more elliptic curve theory g_1, g_2 exist provided the sums in E_{-1} are zero, which we easily check. This means that for the "candidate" $\omega^? = c_1\mathrm{d}g_1/g_1 + c_2\mathrm{d}g_2/g_2$ the difference $\omega - \omega^?$ has no poles. Thus by yet more elliptic curve theory it is $c\mathrm{d}x/y$ for some constant c. With hindsight we can see that $c = 0$, otherwise $\omega = \omega^? + c\mathrm{d}x/y$ would not be elementary integrable contradicting Euler. Or a simple computation using for example

$$g_1 = \frac{1-2y+x^2}{x^2-2x-1}, \quad g_2 = \frac{1-2iy+x^2}{x^2+2x-1}$$

gives $c = 0$ and so there is elementary integrability, which after going back to $y^2 = x^4+1$ yields something equivalent to (4).

But how on earth did Euler do what he did? A later commentary by Hermite [4] doesn't help, because the formulae at the end of his section I say that (4) is an algebraic function of x (of degree 4)!

Families

The argument for (11) shows that if t is a complex number such that

$$\int \frac{\mathrm{d}x}{(x-t)\sqrt{x^3-x}} \tag{12}$$

is elementary then $(t, \sqrt{t^3-t})$ is torsion on E_{-1}. Similarly that if

$$\int \frac{\mathrm{d}x}{(x-2)\sqrt{x(x-1)(x-t)}} \tag{13}$$

is elementary then $(2, \sqrt{-2t+4})$ is torsion on the Legendre curve E_t defined by $y^2 = x(x-1)(x-t)$ (take $t = -1$ to explain the previous notation).

And, with a bit more trouble, that if

$$\int \frac{\mathrm{d}x}{(x-t)(x+t-1)\sqrt{x^3-x}} \tag{14}$$

is elementary then

$$(t, \sqrt{t^3-t}), \quad (-t+1, \sqrt{-t^3+3t^2-2t}) \tag{15}$$

are torsion on E_{-1}. Similarly if

$$\int \frac{\mathrm{d}x}{(x-2)(x-3)\sqrt{x(x-1)(x-t)}} \tag{16}$$

is elementary then

$$(2, \sqrt{-2t+4}), \quad (3, \sqrt{-6t+18}) \tag{17}$$

are torsion on E_t.

Manin-Mumford and the relative case

We can combine the two assertions in (15) simply by saying that

$$(t, \sqrt{t^3-t}, -t+1, \sqrt{-t^3+3t^2-2t})$$

is torsion on the product $E_{-1} \times E_{-1}$ with coordinates x_1, y_1, x_2, y_2. This is essentially the same as saying that we have a torsion point in $E_{-1} \times E_{-1}$ lying on the curve $x_1 + x_2 = 1$.

In the sixties Lang asked how to find all roots of unity ζ_1, ζ_2 satisfying some given non-trivial polynomial relation $F(\zeta_1, \zeta_2) = 0$. Using the standard notation $\mathbf{G}_\mathrm{m}$ for the multiplicative group of all non-zero complex numbers, we see that this is the same as saying that we have a torsion point in $\mathbf{G}_\mathrm{m} \times \mathbf{G}_\mathrm{m}$ lying on some fixed curve. This particular problem was solved very quickly by several people; there are at most finitely many torsion points provided we avoid examples like $\zeta_1^3\zeta_2^5 = 1$ or $\zeta_1^3 = -\zeta_2^5$. It was then natural to replace $\mathbf{G}_\mathrm{m} \times \mathbf{G}_\mathrm{m}$ by any fixed commutative group variety G of dimension at least 2 such as $E_{-1} \times E_{-1}$. This general case of the so-called "Manin-Mumford Conjecture" was treated by Hindry [5] in 1988 (even with curve replaced by algebraic subvariety — but here we stick to curves); again there is finiteness provided we avoid special curves corresponding to group subvarieties.

The situation with (17) is similar but there is a significant difference. We still have a torsion point $(2, \sqrt{-2t+4}, 3, \sqrt{-6t+18})$ but it now lies on the so-called fibre product $G_t = E_t \times E_t$ with coordinates x_1, y_1, x_2, y_2, t satisfying

$$y_1^2 = x_1(x_1-1)(x_1-t), \quad y_2^2 = x_2(x_2-1)(x_2-t),$$

to be considered of dimension 3. Thus the group variety G_t itself is moving with t; an example of the so-called "relative case" of Manin-Mumford. The point lies now on $x_1 = 2, x_2 = 3$, still a curve; but the results of [5] do not apply.

This particular case was treated by Zannier and the author in 2012: again there is finiteness (recently Stoll has shown that there are no t at all).

These examples are needed to handle the integrals (14) and (16). But more generally one has to extend to families — a fancy word is schemes — A_t of abelian varieties of arbitrary dimension generalizing G_t above. For surfaces this was done in a series of papers culminating in [8]. One also has to consider certain non-abelian surfaces which will be mentioned below, and these were treated by Schmidt [11].

Davenport's Thesis

(James) Davenport in his well-known 1981 Ph.D. thesis [1] (Springer "grey series") made precise the idea that "elementary integrability for algebraic functions is rare" by introducing a parameter t; he seems to have done this independently of Manin-Mumford considerations. Roughly speaking, he asserted that if $f(x,t)$ is "algebraic" and $\int f(x,t)\mathrm{d}x$ is not "identically elementary in t", such as

$$\int \frac{\mathrm{d}x}{x^2+t^2} = -\frac{i}{2t}\log\left(\frac{t+ix}{t-ix}\right)$$

(following easily from (2) above), then there should be at most finitely values of t at which it becomes elementary.

Around 2012 Zannier and I had proved some results (see [8] also) on Pell's equation over polynomial rings which seemed to be related to, but not the same as, some key special cases of Davenport's Assertion. As our techniques (see below) were very modern, we were curious about Davenport's proof. Indeed it turned out that this contained gaps. In particular various things like Risch's method, while efficient for transcendental values of t, cannot handle unspecified algebraic values. So we set ourselves the task of constructing our own proof. We confined ourselves to "algebraic coefficients", meaning that $f(x,t)$ is algebraic over $\mathbf{Q}(x,t)$. In fact the presence of a transcendental coefficient seems greatly to increase the difficulty, which may be found paradoxical in view of the remarks above.

One problem concerns the example (12) above. It is easy to see that there are infinitely many values of t such that $(t, \sqrt{t^3-t})$ is torsion on E_{-1}, so the arguments above are inconclusive. With the example (13) it is not so easy to prove the infinitude for $(2, \sqrt{-2t+4})$ on E_t, but here too things are inconclusive. However there is no problem for (14) and (16), because we end up on an abelian surface scheme. Actually this is non-simple, which means that there are many group subschemes, and so these must be eliminated from our enquiries. In fact in higher genus it can be easier; for example with

$$\int \frac{\mathrm{d}x}{(x-t)\sqrt{x^5+x-t}},$$

where one gets a torsion point on a curve inside the Jacobian of the curve

$$y^2 = x^5 + x - t. \tag{18}$$

This curve has genus 2 so its Jacobian is an abelian surface. It turns out to be simple, so there are comparatively few group subschemes to be eliminated. But for example a Jacobian that is the fibre product of two simple surface schemes could also be handled. So somehow it is elliptic curves that are making all the trouble. And indeed after generalising the torsion results on [8] to arbitrary genus $g \geq 2$, we could verify Davenport's Assertion whenever there is no surjective map from C_t to an elliptic curve E, where C_t is the underlying curve as in (7) and (18). This amounts to saying that the Jacobian J_t of C_t has no elliptic factor.

Torsion on additive extensions

If there is a surjective map from C_t to E, as in (12), then comes another key step, apparently no longer classical, in that we must consider the zeroes of the differential $\omega = f(x,t)\mathrm{d}x$. Let us look at (12) more closely, with a double zero at O.

As after (11) we get $(g_i) = k_i^+(P_t^+ - P_t^-)$ $(i = 1,\ldots,m)$ now for $P_t^\pm = (t, \pm\sqrt{t^3-t})$. It follows that the multiplicative rank of $g_1,\ldots,g_m$ modulo constants is at most 1. This contradicts the minimality of m unless $m = 1$. We can assume $g_1(O) = 1$, and now $\omega = c_1\mathrm{d}g_1/g_1$ shows that $g_1 - 1$ has at least a double zero at O.

We have arrived at an example of a "generalised Jacobian" as introduced also by Rosenlicht: the group of divisors (of degree zero and with zero multiplicity at O) modulo principal divisors (g) such that $g-1$ has at least a double zero at O. It is a commutative group variety G (not projective like an abelian variety), and in fact we have an exact sequence

$$0 \longrightarrow \mathbf{G}_\mathrm{a} \longrightarrow G \longrightarrow E_{-1} \longrightarrow 0 \tag{19}$$

where $\mathbf{G}_\mathrm{a}$ is the additive group of all complex numbers. The projection to E_{-1} comes from the standard equivalence relation on divisors forgetting the double zero at O. The embedding of $\mathbf{G}_\mathrm{a}$ comes from taking z to $(1 + zx/y)$, for example, because

$$\left(1 + z_1\frac{x}{y}\right)\left(1 + z_2\frac{x}{y}\right) - \left(1 + (z_1+z_2)\frac{x}{y}\right)$$

has at least a double zero at O.

Thus the class of $P_t^+ - P_t^-$ is torsion on the additive extension G. Such things seem not to be in the literature on elementary integration.

It is crucial here that G is non-split; i.e. not isomorphic to $\mathbf{G}_\mathrm{a}\times E_{-1}$, because then there are fewer group subschemes and [11] yields the required finiteness.

In this way we could verify Davenport's Assertion when there is a surjective map from C_t to E, but only when E does not have complex multiplication CM (which would also lead to more subschemes). However it is necessary to use extensions more complicated than (19), and actually products of such things, even fibre products, and furthermore certain quotients of them. Now verifying non-split is a much more technical matter, not really in the previous literature.

At several stages of this part of the investigation we came up with integrals that looked like counterexamples to the Assertion; but each one was shot down, partly with the aid of computation (see below).

Note however that E_{-1} in (7) does have CM, with $\mathbf{Z}[i]$ as the endomorphism ring (thus $i(x,y) = (-x, iy)$ for example). Our methods worked for CM, but only when $m \neq 2$ in (8). We left the CM case $m = 2$ until the very last because we were sure that it would be easy. And indeed we could handle things like $\mathrm{d}x/((x^2-t^2)\sqrt{x^3-x})$ with various dirty tricks. However all these tricks failed for

$$\omega = \frac{x\mathrm{d}x}{(x^2-t^2)\sqrt{x^3-x}}.$$

All we could deduce from elementary integrability was that the point $P_t = (t, \sqrt{t^3-t})$ on E_{-1} must be torsion, rather as for (12).

For an account of much of this see Zannier's ICM talk [12].

The final problem

At this point we resorted to our tried-and-tested "candidate construction". Choosing t with $nP_t = O$ for some fixed $n \geq 1$, we find $\overset{?}{\omega} = c_1\mathrm{d}g_1/g_1 + c_2\mathrm{d}g_2/g_2$ if ω is elementary integrable. As before this means $\omega - \overset{?}{\omega} = c\mathrm{d}x/y$ for some $c = c(t)$. In all the earlier work we always found numerically $c \neq 0$, and since $\mathrm{d}x/y$ is never integrable this means that ω is not.

We started off with $n = 4$ and $P_t = (i, 1-i)$. We found $c = 0$, unusual but it could have been a coincidence with the endomorphism ring being $\mathbf{Z}[i]$. So still with $n = 4$ we took $P_t = P^+ = (1+\sqrt{2}, 2+\sqrt{2})$. And still $c = 0$!

Now a bit worried, we took $n = 3$ and $P_t = (\frac{1}{3}\sqrt{9+6\sqrt{3}}, ?)$. And again $c = 0$!!

Now somewhat perturbed, we made the jump to $n = 5$ and $P_t = (\frac{1}{5}\sqrt{5+10i}, ?)$. But still $c = 0$!!!

It was then that we realised we had a counterexample. It took another hour to prove it, and indeed that ω is elementary integrable for any t such that $nP_t = O$ for some $n \geq 3$ (and no other t).

And the explicit integration for $n = 5$, perhaps to be compared with (4), is

$$\int \frac{x\mathrm{d}x}{(x^2 - \frac{1}{5} - \frac{2i}{5})\sqrt{x^3 - x}} = c_1 \log g_1 + c_2 \log g_2 \quad (20)$$

with $c_1 = 1/2b,\ c_2 = -i/2b$ for $b = \sqrt[4]{220 + 40i},\ a = -(2+i)b^2/10$ and g_1, g_2 are

$$-\frac{10axy - (15-5i)bx^2 - 50y + (2-4i)abx + (3+i)b}{10axy + (15-5i)bx^2 - 50y - (2-4i)abx - (3+i)b}$$

$$-\frac{10aixy - (15-5i)bx^2 + 50iy - (2-4i)abx + (3+i)b}{10aixy + (15-5i)bx^2 + 50iy + (2-4i)abx - (3+i)b}$$

(with of course $y = \sqrt{x^3 - x}$) respectively, which Maple 18 cannot check even by differentiation due to the complicated number field involved (however it can check equality up to say 1000 decimal places when we integrate between say $x = 2$ and $x = 2.1$, and that is somehow thoroughly convincing).

The integral for $n = 7$ would involve an algebraic number of degree 12 in the denominator of the left-hand side of (20); but g_1, g_2 might no longer fit into one line (even of normal length).

In the end we were able to describe all counterexamples to Davenport's Assertion. They are uncommon, which after all does indeed support the original idea that "elementary integrability for algebraic functions is rare". A way of eliminating them altogether is to restrict the values of t to some fixed number field or even to those with a fixed upper bound for the degree $[\mathbf{Q}(t) : \mathbf{Q}]$; in that case one gets finiteness without any counterexamples. The proof of this is relatively easy using a height boundedness theorem of Silverman (1983),

Apart from the features already mentioned, the main techniques of our proofs are: a Galois-theoretic result of David (1991), proved by transcendence techniques, a functional algebraic independence result of André (1992), proved by Hodge techniques, and a counting result for subanalytic surfaces of Pila (2004).

The integrals

$$\int (\log x)^t \mathrm{d}x = (-1)^t t! x \sum_{m=0}^{t} (-1)^m \frac{(\log x)^m}{m!}$$

for $t = 0, 1, 2, \ldots$ show that bad counterexamples to finiteness for non-algebraic functions exist, and are much easier to find.

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David Masser

David Masser FRS retired from the University of Basle at the statutory age of 65, and maintains a keen interest in various aspects of diophantine geometry especially regarding the application of techniques usually associated with transcendence theory.

An Interview with Simon Tavaré

JUNE BARROW-GREEN AND VICKY NEALE

Professor Simon Tavaré FRS, FMedSci, is the outgoing President of the London Mathematical Society. He is the Director of the Cancer Research UK Cambridge Institute, and is Professor of Cancer Research (Bioinformatics) in the Department of Oncology and Professor in the Department of Applied Mathematics and Mathematical Physics at the University of Cambridge. We asked him about his mathematical life and work, and his reflections at the end of his two-year term as President of the LMS.

Simon Tavaré (Photo courtesy of Charles Thomson)

JB-G Can you tell us what first sparked your interest in mathematics?

I suspect, as with almost everyone who doesn't start at a precociously young age, it would be a teacher. Mine, at age 15, was Chris Jones, who went on to be head of mathematics at Eton. I was also good at languages, and that too was primarily because of inspirational teachers.

VN So what did he do that inspired you, or what did he share with you?

I'm not sure I can put my finger on that. Like most people, I wasn't very sure when I was a kid what I was good at. I was streamed when I was about 13, far too young. I was put in the "future scientist group", and in the following years we focused on science and maths, including projective geometry and probability theory, generating functions, calculus, all sorts of things, it was quite sophisticated as I later learnt. I was in a class with some quite talented people. One or two of them were extraordinarily smart, the precocious maths types. I wasn't one of those. I ended up in the science track, mainly because of the maths part and encouragement from Chris Jones. Lots of calculus ... I used to be able to integrate anything when I was 15; now there's no way I could do it. Not that I have to, but it would be hopeless. This was pre-computer algebra so you had to do it yourself, all those transformation tricks. I do remember getting a school report at the end of my school career that said something like "This boy obviously understands nothing about mathematics and should be encouraged to do something else." I thought that was hurtful. But it was probably true.

VN Was it then easy to choose to follow maths further, or were you torn between that and other options?

Effectively, I had two years off between school and university. I was a bit too young to go to university straight after A-levels. I decided to stay on at school for another year. This was a nightmare! I then had another year off; I was a garbage man for six months, and an actuarial student for six months. The actuarial job convinced me to do maths at University, because I enjoyed doing it.

JB-G Having completed your undergraduate degree at Sheffield, how did you choose your area for a PhD?

As an undergrad, I was particularly interested in probability and statistics. I was a sporty type, and I played with some biology PhD students. A lot of them had data they couldn't understand anything about, so I used to help them do what was really entry-level data analysis. So I thought statistics would be an area to explore further.

JB-G Was it the maths in it, or was it the application of it that interested you?

Both, actually. I thought that the probability part was very pretty. It was very fingers and toes and counting things. It uses lots of clever arguments that try to avoid hard analysis. That's been a theme of lots of the things I've done. I like probabilistic tricks (intuition?) that get fast answers to hard problems. Definitely the probability part was the main bit, but as it was set up then in Sheffield there was a Department of Probability and Statistics. It was about the best group in the world, but of course as an undergraduate you are not aware of that. I stayed in Sheffield as a research student, not by choice really, but for family reasons. Later on, when I got to be a postdoc, I worked with the person I originally wanted to be a student with at Stanford, so it worked out OK.

My work has been in both probability and statistics. Of course these have intersections, which in my case is somewhere around the theme of inference for stochastic processes and how you fit models to data. That's something I've done for a long time. The other thing is that a lot of this applied stuff is rather opportunistic, and this is certainly the case now. Sometimes new data types turn up and then you have to think about what you're going to do about them: do they need new theoretical methods, or adaptations of existing ones, and so on. So we do a lot of that sort of thing now, again motivated by data, but with principled methods of analysis in the background. We try to do this with a combination of stochastic processes, statistics, inference and real biology. I am not one of those mathematicians who believe that mathematical biology has nothing to do with biology!

My PhD supervisor, Chris Cannings, is a direct descendant of R.A. Fisher, academic-wise, and so we were interested in problems in genetics and mathematics. At that time there were few data around, so it was a very theoretical discipline, even though in fact it made predictions that worked beautifully when the data did turn up. So we got into stochastic process problems motivated by genetics. That turned into a big field very quickly, as soon as people started collecting large amounts of DNA data. We used these methods to extract information about population history from DNA data people collected. Suddenly the field went from having no data to having infinitely much, and now of course it's one of the most active bits of science there is.

VN When you started your PhD, did you have to learn a lot of biology, or had you already done that?

I'd already done a lot of that. At that time we were not trained to be both biologist and mathematician, but you did at least have to have an interest in the biological principles as understood, so that you had a chance of focusing on scientifically reasonable problems. Nowadays I see postdocs with very strong backgrounds in both disciplines, and that is a great improvement.

The US tenure system, at the time I was going through it, was tough for people with what we now call interdisciplinary interests. To get a job in a department you had to have a good publication record, in an area that's yours, and if it was a maths department then you did something mathematical, and it could be motivated by whatever you liked. Physics is the obvious example but it could be biology, chemistry, or whatever. But they were really judging the mathematics. And that, I think, is changing, thank heavens. People now get recognition for doing both.

JB-G How do the problems emerge? Is it by having discussions with biologists?

Yes, by connecting with other quantitative science people. The genomics field is now very, very much bigger than it was when I started. We were really at the forefront of this, because DNA sequencing only turned up in 1980. At that time the focus was on understanding the genetic behaviour of idealised populations, or real ones but of model organisms like fruit flies. There were a lot of fly genetics groups around the world that worked out the biology of all sorts of things which were later discovered to be true in humans. The theory came from population geneticists, in particular Fisher, Haldane, Wright and Kimura. Mathematics had a key role to play in this, as Kingman showed in the early 1980s. His three coalescent papers in 1982 changed completely the way that population geneticists understood natural variation.

VN How easy is it to get your work published in an appropriate journal? Has it become easier in the last 35 years as this field has grown?

No, I don't think it has. What I do now, what I've done for the last few years, is really the biological end. We're trying to help the biologists understand their data in a principled way, so it's all statistics in some way, or probability, but the questions are very definitely front-line cancer research. What we decided in our group, rightly or wrongly, was to publish the science in the best places we could, and if the maths is in the appendix, that's how it goes. We're still on the paper, and it's in *Science* or *Nature* or whatever. If you want your work to be read by people in that field, you have to put it in their journals, you have no choice. They are not going to read maths journals of any sort, sadly.

The scale of the biological research field is much bigger than the mathematical one, probably by a factor of a hundred. The way their citations work and the way they get recognised is completely different from mathematics, and they're judged on completely different things. This is in part reflected in the non-alphabetical lists of authors!

Biologists are also early adopters of technology. This can be very tricky for students, from a mathematical perspective; if somebody comes out with a new technology for measuring something, last week's technology is gone. So if your student, and I've had this happen to some of mine, is working on some novel computational technique for analysing a particular sort of data, all of a sudden these data aren't going to get produced. You might have done a lovely piece of work, and it's OK for a thesis, but it's never going to get used. I guess that's the nature of the animal. So you have to get this mixture of principles which you could use in different settings, which is of course the mathematical tradition, and then you have to take a punt on getting something done fast enough to be early in the list of methods developers, because if you're fifth or something nobody will ever use it either. It's a very delicate position.

VN That sounds quite precarious for a graduate student or a postdoc.

For postdocs it's a bit easier, because by then they usually have enough nous to know where they're headed, or where they don't want to go. For PhD students it is hard, because they all go at different speeds, and they might be working in interdisciplinary groups with medics and so on. It is a problem. I've had some students who accept this and thrive, and some who do it for a PhD and then change fields, because they just can't take the uncertainty of it all. I've had some stunning maths students, and one or two of them are happy with this sort of rather messy, data-driven sort of thing, and one or two of them just could not stand it. We also have a wet lab, so we can generate our own data, in part because we didn't want to have our students fighting for possession of little bits of a huge international project. Each student has their own data to look at and to think about, and if they want to do more experiments they can get them done, and some students make a real good show of that, they actually do originate new experiments. That's really good fun to watch.

JB-G Do these students have any biological background?

Some of mine do, and now it is more typical for PhD students to get put through some courses that give them background information. You've got to know a little bit as you've got to be able to talk to your collaborators. And that's hard. One of the problems is that mathematicians are often quite shy. We don't ask a lot of questions in lectures in England for example, whereas biologists are asking questions all the time. They are fearless, they don't care whether the question's silly, eventually they'll get the details sorted out in their heads, and that's what they want. Part of the art of collaboration is communication, of course, and that's very hard: you can't really teach it.

JB-G I imagine that the public would be quite surprised by the number of mathematicians working in your cancer research institute.

Absolutely. There are about 80 quantitative people in our institute, including computer scientists, statisticians, bioinformaticians and mathematicians. Some are quite applied, some come from the more IT end of the computer science world. Many of them have maths degrees of some form or other, and some have converted from wet lab biology. That's out of about 420 scientists. The number of quantitative people is growing rapidly, primarily because of the data-driven angle of science, and we are a particularly big scale data-driven place, with imaging and DNA sequencing for example. The imaging systems now are just stunning, and the amount of data they produce is eye-watering. So now there's a role for genuinely clever deep learning approaches for big scale data analysis problems, which are gradually cranking up in the DNA and cancer research world.

VN Could you say a bit more about collaboration in your work?

Of course mathematics is often done in collaboration, and there have been some extraordinary mathematical collaborations typified by Hardy and Littlewood for example. I think now it's a bit more customary to have papers that are written with more than one mathematician on them. In physics, astronomy, there are papers with hundreds of authors, and it's true in biology too that papers have a lot of authors on them.

At some level it boils down to how you hand out the credit. I've written many papers in two collaborations over the last 30+ years, the first with Richard Arratia and Andrew Barbour, and the second with Bob Griffiths. We still work together occasionally and we use Littlewood's rules of collaboration. Somebody sends round a first draft of a paper and everyone's on it in alphabetical order, it doesn't matter who did what. This works fantastically well, no recriminations. So that's the fun part of mathematics I think.

But who gets the credit and how it's decided is much more difficult in an interdisciplinary world, especially in the medical area, because usually the quantitative people are in the middle: the head of the lab is on one end of the list of authors, and the chief postdoc is on the other end, and there's a bit of a fuss around 2 3 4 5 and 108 109 110. And then the problem comes when they're evaluated, it's "oh they're just middle authors". I think senior scientists have to write careful evaluations, to explain the crucial role played by middle authors. I know that if my group hadn't done the analysis there often wouldn't be a paper, and yet we're still in the middle. So it's a tough game. What I hope to be able to train them to do is not to worry about where they are on the list. My advice is to make sure you've done something which is justifiably decent. You can always write a methodology paper, which probably won't get cited very much, but you should do that for the scientific record. One of the problems with these big mixed papers is you don't get a proper track record of what happened: the methods are often not written up properly despite people saying they are. Things are then not reproducible.

JB-G Are there many women working in your field?

Yes. Well, in biological sciences at PhD level it's more than half women. Where it tails off is the same as everywhere else, which is at group leader level, and at professor level. At the moment, I think our breakdown is 52% women in the whole institute. I'm sure among the quantitative people there are fewer women. I think I have maybe half at the moment, or a bit less than half. But that's not the usual input into quantitative PhD programmes, of course. I run a Wellcome Trust programme, which is a mixture of mathematics and genomics, and that has very few women.

VN How has it been being President of the LMS?

I hadn't realised how short two years are. I was hoping we'd have an established LMS fundraising system in place, but what I hadn't realised when I started is that it's not uniformly appreciated as something we should be doing. This always puzzles me, because, all things being equal, it's a lot easier to do things if you have a bit more money than if you have a bit less. That hasn't gone how I had hoped it would. We came very close to some interesting contributions, and then we failed at the last step or two.

I do think that fundraising is potentially a difficulty for the LMS; if the publishing business changes much more, I can't see it continuing to be as profitable as it currently is, and this will certainly impact our ability to support mathematics and mathematicians across the UK.

VN Why should young mathematicians join the LMS?

Most of our effort goes to supporting young mathematicians in various ways. However, you don't have to be a member to get those benefits. I would hope, though, that contributing to the LMS by joining, and playing a role in the way it supports the discipline, would be attractive. Of course, the stresses of modern academic life, be it family or publishing one's own work, do mitigate against contributing to the way a society runs, or reviewing, or journal editing or the myriad other things we get asked to do. There are choices to be made. I would like to encourage younger mathematicians to apply for roles on the LMS Council. It is a way to get your opinions heard, and you have a chance to influence the way one of the world's great mathematical societies operates. Put another way, if you don't like what you see happening, try to change it!

JB-G How do you see the role of the LMS in the wider mathematical community?

The LMS has pitched itself as the society for professional mathematicians, or arguably pure mathematicians, so it probably doesn't engage quite as far down the education spectrum as it might. I know lots of very good pure mathematicians who work in all sorts of areas of science. They're not quite so protective of their turf, they realise that mathematical arguments apply all over the place, and if we could represent that more I think that would be fantastic. We've certainly tried. There's a joint meeting coming up with the IMA, which looks very interesting — I just signed up.

Of course there is a perceived problem with our public face. I don't mean the society, I mean mathematicians as a whole, who are often viewed as some sort of rather nerdy back roomy sort of people. I think this is an image problem that nobody really knows how to fix. The statisticians seem to be much more visible for some reason. It's challenging to explain to people why you want to be sitting all weekend working on some combinatorial problem that's difficult. So it's not the LMS really, it's how mathematics gets itself into the wider community. Having such small numbers of people doing mathematics after age 16 is a problem, and we also have this attitude, which you'll never hear in France, of proudly saying "oh I was no good at maths when I was at school".

JB-G What advice would you give to a young mathematician?

I learned a long time ago never to give advice. I do think independence is crucial, but not always something we stress as PhD advisors. I would also encourage PhD and postdoc advisors to get their mentees to explore other options where mathematics has a real role to play. There are many! I've had 40+ PhD students, and obviously there aren't forty something professorships when I go, there's one. So I would encourage people not to think that if you don't get a lectureship somewhere you're a failure, I think that's a disaster for mathematics.

Another thing I think is worth doing is changing fields occasionally. You can't do this too often, of course. Even staying in the same field but moving universities is worthwhile. A change of environment, scenery, colleagues, can make things much more interesting.

VN Was it always clear to you that you wanted to follow an academic trajectory, or did you ever think about doing other things?

I come from a business-oriented family. My dad, who was knighted for his founding contribution to the Mersey Basin Campaign (which he did after retiring), wanted me to be a merchant banker. My middle brother followed in the business vein, and the youngest one is a very well-known comedian. When my dad was ninety something he still kept asking "when are you going to get a proper job?", which I still giggle about. I think he never really understood why someone would take a job at a fifth of the going rate for bankers. We're not all driven by the same things of course ... and he was very proud when I was elected to the Royal Society!

I still marvel at the fact that as academics we are free to explore what we really enjoy. We work in an international maths and science community and develop international collaborations. Despite spending the last twenty years in the cancer world, I still have very good mathematical friends, and we still write papers together, which is fun. One of the things I got out of being President of the LMS was getting back into things mathematical; it is easy to get sidetracked by more administrative responsibilities. That's why I gave it five years at the CRUK institute. It's been fun, in the sense that being President of the LMS has been fun; you learn a whole load of new things, you meet lots of new people with different backgrounds.

JB-G What would you say has been your greatest achievement in your time at the CRUK Institute?

I made some fantastic hires, absolutely world class. Also, something much more difficult to put your finger on. It's not just responding to Athena SWAN, it's much more than that. We've made the place a really employee-friendly place to work; people actually want to be there. We've spent a lot of time working on that, that's been very good. We have made some major contributions to cancer research. And I do think now we're CRUK's best science institute.

JB-G You mentioned that the LMS got you back into things mathematical. Could you elaborate?

In my presidential lecture in November I'm going to connect several former LMS presidents together — they all worked on related problems, which I am working on as well. It's motivated by the following simple problem. Take n strands of cooked spaghetti. Pick the end of a strand, randomly choose another end and tie a knot, so you're making spaghetti loops. Keep going until you run out of spaghetti. What's the chance all the lengths are different? What's the length of the longest loop? De Morgan was, implicitly, working on a version of this problem. It's to do with a combinatorial distribution that plays a role analagous to that of the normal distribution in the central limit theorem: put briefly, "lots of combinatorial objects look like this one". This distribution is called the

Ewens Sampling Formula, and appeared in 1972 in a celebrated genetics paper by Warren Ewens.

VN Do you have a favourite mathematical book?

My favourite mathematical book, a bible for starting probabilists, is Feller Volume 1. Feller wrote two superb textbooks that every probability student reads. Volume 1 is full of counting problems, and goes on to introduce all sorts of stochastic process theory. A very nice book, and still a good read.

JB-G Do you have a favourite mathematical result?

It's another of these combinatorial things. Erdős and Turán proved this wonderful, if odd, result: take a random permutation, and multiply it by itself until you get back to where you started. The number of multiplications required is the order of the element. Their question was, what's the asymptotic behaviour of the order of a randomly chosen n-permutation? The beauty of this thing is they proved this result which says the log of the order is asymptotically normal, with mean $(\log^2 n)/2$, and variance $(\log^3 n)/3$. Now where on earth did that come from? They wrote a paper about the result, I think it came out in 1967, and it was typical Erdős. It involved hard, nasty calculation. At the end they said "Our proof is a direct one and rather long; but a *first* proof can be as long as it wants to be. It would be however of interest to deduce it from the general principles of probability." Richard Arratia and I, and others, have since provided rather shorter proofs of this result. Ours exploited probabilistic tricks relating to the Ewens Sampling Formula theory that don't use any analysis whatsoever. I'm proud of that one, and it's a lovely result, because it's totally unexpected why it looks like this.

VN What are your chief interests aside from mathematics?

I really do care about cancer research, and I hope my group's work is providing useful methods for interpreting the deluge of data we now get from cancer studies. On the home front, my *Who's Who* entry noted an interest in fine tequila — till they deleted it.

JB-G and VN Thank you very much!

June Barrow-Green

June Barrow-Green is Professor of History of Mathematics at the Open University and Visiting Professor at the London School of Economics. She is also the LMS Librarian. When she isn't teaching mathematics or buried in an archive, she is either running marathons or on a bicycle.

Vicky Neale

Vicky Neale is the Whitehead Lecturer at the Mathematical Institute and a Supernumerary Fellow at Balliol College, Oxford. She spends most of her time enthusing about mathematics to undergraduates, school students, members of the public, and her two cats.

Reciprocal Societies: The Danish Mathematical Society

Founded on 8 October 1873, The Danish Mathematical Society (DMF) is one of the oldest national mathematical societies (preceded and presumably inspired by the mathematical societies of London (1865), Finland (1868) and France (1872)). The purpose of the society was and still is to promote research and teaching in mathematics. The foundation of the society is generally attributed to T.N. Thiele, Hieronymus Georg Zeuthen, and Julius Petersen.

T.N. Thiele

Soon after its foundation the society had attracted 65 members consisting of a widely mixed group of mathematically interested individuals, ranging from military personal, through elementary school teachers, to university professors. For example Johan Ludwig William Valdemar Jensen (known e.g. from Jensen's Inequality) worked as an engineer for the Copenhagen Telephone Company and served as president of the society 1892–1903. Jensen was succeeded as president in 1903 by Vilhelm Herman Oluf Madsen, who was the Danish Minister of War 1901–1904. Subsequent presidents of the society include the influential Danish mathematicians Harald Bohr and Børge Jessen.

In the early days (and until the 1930s) the society was strongly concentrated in Copenhagen, as was the case with Danish academia in general. The society would have a meeting approximately once a month, where a mathematical issue was presented and debated. In the first half of the 20th century the DMF was simply the centre for high level mathematics in Denmark, featuring lectures by some of the world's most prominent mathematicians (Hilbert, Hardy and Lebesgue just to mention a few).

Beginning in the 1930s local universities were founded outside Copenhagen, and eventually this also entailed new Danish mathematics departments. As of 2017 there are six mathematics departments in Denmark (at the universities of Aalborg, Aarhus, Southern Denmark, Roskilde, Copenhagen and at the Technical University of Denmark). The number of university mathematicians working in Denmark increased rapidly during the period 1950–1980, but has been relatively stable since the 1980s. In the same period the number of members of the DMF increased correspondingly and the present number is around 350 members. The profile of the members has also changed so that most present members are affiliated with university mathematics, whereas maths teachers from elementary schools and high schools have formed independent associations.

While the decentralisation of Danish academia has had an increasing effect on the number of DMF members, it has also radically changed the nature and frequency of the activities of the society, since the members are now scattered around the country. Even in a small country like Denmark it has proven difficult to attract members from distant parts of the country to meetings and lectures in the society. It is therefore the present strategy of the society to concentrate attention to a few high profile meetings and seminars per year with the ambition of significant member participation. Annual lectures and associated arrangements with the Abel Prize winners constitute one of these highly profiled future activities of the society. Another is the celebration of the winner of the DMF prize for the best Danish masters thesis in mathematics. This prize has been awarded annually since 2007 and is sponsored by the private company Edlund A/S. It remains a major point of focus for the Danish Mathematical Society to serve as a unifying network for all (high-level) mathematics in Denmark and as a common mouthpiece for Danish mathematicians in national and international mathematical matters.

A comprehensive account on the history of the Danish Mathematical Society (from which the text above has borrowed a lot) is Kurt Ramskov, The Danish Mathematical Society through 125 Years, Historia Mathematica 27 (2000), 223–242.

Steen Thorbjørnsen
President of the Danish Mathematical Society

Editor's note: the LMS and the DMF have a reciprocity agreement meaning members of either society may benefit from discounted membership of the other.

Member Consultation: How Can the LMS Broaden its Support for Diversity in Mathematics?

EUGÉNIE HUNSICKER

The London Mathematical Society strongly supports advancing women's careers in mathematics. Through its Women in Mathematics Committee, formed in 1999, and through the Good Practice Scheme, it engages in a variety of activities aimed at supporting women in their careers and helping departments to embed equal opportunities for women within their working practices. The society's efforts on behalf of women in mathematics were recognised last autumn with the award of the first Royal Society Athena Prize for contributions to diversity in science. The LMS is committed to continuing its work to support women in their mathematical careers, and is now considering if and how to broaden its efforts at supporting diversity within mathematics. The issue has been discussed both within the Women in Mathematics Committee and within Council. The decision was made to consult with the LMS membership about the diversity issues beyond that of women in mathematics that it would like to see addressed, and also to solicit suggestions of how this might best be done through an article in the Newsletter (this one!).

Women in mathematics

The LMS has a wide range of activities to support and promote women in mathematics, that may serve as some inspiration for possible activities in other diversity areas (though of course we also welcome other ideas!). These activities can be broken into three general categories.

The first category is *celebration* of the work of women in mathematics. This is seen through the annual Mary Cartwright Lecture, which showcases the work of an eminent woman mathematician, and the Anne Bennett and Senior Anne Bennett prizes, awarded either for work by a woman or work to support women in mathematics.

The second category is *support*. This is seen through the Grace Chisholm Young Fellowships, given to support individuals who have had a break in formal employment so they can resume a career as a mathematician afterwards; the Caring Supplementary Grants, awards to enable mathematicians with caring responsibilities to participate in conferences or research visits by partially offsetting additional caring costs; and the annual Women in Mathematics workshops, open to all mathematicians, but particularly celebrating the work of women in mathematics and aimed at creating networks for young women in mathematics.

The final category is *advocacy*, seen through the work of the Good Practice Scheme and the Benchmarking Survey, which was launched at the House of Commons.

Broader diversity issues

Broader diversity categories may include those aspects of diversity that are protected under the Equality Act of 2010: age, disability, gender reassignment, race, religion or belief, sex, sexual orientation, marriage and civil partnership, pregnancy and maternity. But they may also include other aspects of diversity, such as neurodiversity, transgender or nonbinary gender identity, national background, nontraditional career path, paternity, socioeconomic diversity or others that may be of interest and concern to you.

The LMS is eager to hear from its membership, about your thoughts on these issues and what the LMS might consider doing to better support all mathematicians. Please email me, E.Hunsicker@lboro.ac.uk, with your thoughts, which I will collate and bring to Council. Thanks very much for your input!

Eugénie Hunsicker

Eugénie Hunsicker is the Chair of the LMS Women in Mathematics Committee and sits as the LMS representative on the Athena Forum. She is a senior lecturer in mathematics at Loughborough University with current research interests in high dimensional statistics. She has three teenage sons, a chihuahua and two enormous rabbits and has been known to torture the LMS Council with her singing and ukulele playing.

Success Stories in Mathematics

What does it mean to be a successful mathematician? What is involved in a successful mathematical career? The LMS Success Stories project aims to celebrate the diversity of successful careers and mathematicians. We are always interested in new profiles! If you have an idea, or would like to submit your own profile, please email Success.Stories@lms.ac.uk.

Name: Hua Lu
Job: Research scientist, British Antarctic Survey

I'm originally from the south-west of China, where my favourite activities were swimming in the Aha Lake and wandering miles and miles in the mountainous countryside. At the time, I was also a headstrong teenager who rejected the dogma that boys are better at maths than girls. Putting maths as my first choice of tertiary education was an emotional decision. After several years of struggle in learning pure maths in SiChuan University and Operational Research in Guizhou Institute of Technology, I started to develop a thinking mode that combined rigorous formalism, intuition and creativity. This ability has benefited me since then. It built the foundation for me to become a passionate scientist.

Trained as a mathematician, but seduced by Mother Nature's beauty, I turned my focus onto environmental research. I moved to Australia in 1993 to study for a PhD in applied mathematics at the University of New South Wales. I took a challenging project that involved mathematically modelling dust emission and long-range transport of atmospheric particulates. It was my first taste of the real world application of maths. I juggled with extensive datasets and collaborated with some brilliant researchers. My confidence and passion grew.

After my PhD, I worked for the Land and Water Division, Commonwealth Science and Industry Research Organisation (CSIRO) in Canberra, as a postdoc and subsequently as a research scientist. Ten years later, I relocated to Cambridge with my family to escape bush-fire flames and to enjoy the smell and scent of green at the Backs. I was a visiting fellow at the Institute of Theoretical Geophysics, Department of Applied Mathematics and Theoretical Physics, University of Cambridge in 2003–2004. I have been a research scientist at British Antarctic Survey since 2005.

The rigour of maths that I gained earlier shines light on all of my research activities. It allows me to tackle environmental issues rigorously with an interdisciplinary approach. I have worked on wind erosion and dust transport, sediment transport within river basins, solar influences on Earth's climate variability, and most recently on stratosphere-troposphere coupling. Climate is at the core of all the research issues I have encountered so far. It is the most exciting environmental science because the system is apparently simple but rather complex, so natural but deeply mysterious. It changes constantly and one can hardly have a thorough grasp of its shape. My recent research interest is on investigating how perturbations imposed in the middle and upper stratosphere can be transmitted downwards to the lower stratosphere and the troposphere dynamically through wave mean flow interaction. One example of this kind is the identification of multiple solar influences and pathways in the climate records, studying non-linear amplification of solar signals and understanding dynamic interactions between external forcing and atmospheric internal variability from a perspective of whole atmospheric vertical coupling. Again, my maths background benefits this new adventure enormously.

In my spare time, I enjoy practising Tai Chi, playing badminton and gardening.

Microtheses and Nanotheses provide space in the Newsletter for current and recent research students to communicate their research findings with the community. We welcome submissions for this section from current and recent research students. See newsletter.lms.ac.uk for preparation and submission guidance.

Microthesis: Mathematical Models for Glass Sheet Manufacture

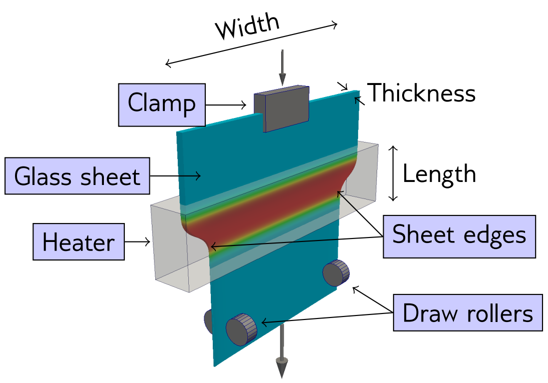
DOIREANN O'KIELY

The manufacture of touchscreens and other slim modern devices presents a challenge: how can we create very thin sheets of glass with precise specifications? Mathematical modelling aids in the design of a new manufacture technique by explaining the flow pattern of molten glass during production.

Glass sheets in modern life

The touchscreens and cameras in smartphones and other modern devices rely on thin glass sheets. The drive for these sheets presents new challenges for manufacturers: they must be tens of micrometres thick, completely flat, and free from thickness non-uniformities, which cause optical distortions.

Glass sheet redraw

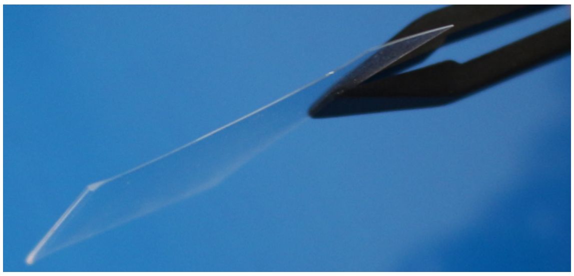


Schematic of the glass redraw process (Reprinted from [1] with permission.)

In the redraw process, a preformed glass sheet is fed into a heater where it is melted, and then pulled out of the bottom of the heater at an increased speed, resulting in a thinner final sheet. As the molten glass is stretched, its cross-sectional area must decrease. This may be accommodated by a change in thickness (thinning), and/or a change in width (lateral necking). The interaction between thinning and necking may lead to undesirable thickness non-uniformities.

The challenge

Glass sheets produced using the redraw process are typically thicker at the edges than in the centre. Out-of-plane deformation of the sheet centreline is also observed in some cases. These imperfections cause optical distortion, and also mean the sheet is likely to break during post-processing.



Redrawn glass sheet with thick edge

The mathematical challenge is to develop a model that describes the glass motion during redraw, use it to explain the source of imperfections, and ultimately investigate mechanisms for avoiding these imperfections.

A glass sheet undergoing redraw may be modelled in a relatively straightforward way by considering a highly viscous liquid sheet, and exploiting the fact that it is very thin to derive a system of differential equations in two spatial dimensions: length and width.

Glass thickness

When the edges of a wide sheet move inward, the resulting sheet is thicker at the edges than in the bulk. We may think of this as glass being "gathered" by the edge as it moves towards the sheet centre. If the molten portion of the sheet is very long, then this glass may be redistributed across the sheet, yielding a uniform thickness. In reality there is not sufficient time for the molten glass sheet to reach uniform thickness, and the "thick edges" are frozen in.

Solving the governing equations numerically allows us to determine the thickness profile of the sheet, and also to identify a modified preformed sheet, which is thinner at the edges and will redraw to a product with uniform thickness.

Model parameters

Analysing the model for glass redraw reveals two key parameters: the ratio D between the final and initial thickness and the ratio δ between the length of the heater and the initial width of the sheet.

In the asymptotic limit $\delta \gg 1$ (when the heater is long compared with the sheet width), the width and thickness decrease in the same proportion, by a factor $1/\sqrt{D}$. Thickness non-uniformities are given by higher-order corrections to this solution, and scale with $1/\delta$.

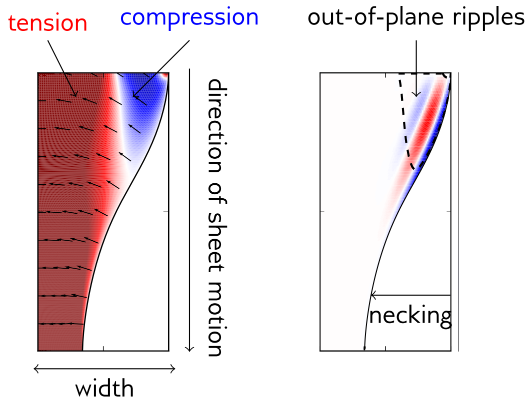
However, in the limit $\delta \ll 1$ (short heater), the thickness decreases by a factor $1/D$ and the width only decreases by a small, $O(\delta)$, amount. The small change in width and resulting accumulation of glass gives rise to a different scaling near the sheet edge, where the thickness decreases by a factor $1/\sqrt{D}$.

Out-of-plane ripples

Tension imposed by rollers causes the glass sheet to accelerate as it travels through the heater zone, but in the lateral direction there may be tension *or* compression, and this may vary in different regions of the sheet. For example, the sheet typically experiences lateral compression near its edge in the upper part of the heater, where the edge is beginning to neck inward.

In regions where the sheet experiences compression, it will buckle. This leads to out-of-plane ripples in the molten sheet, and if these ripples are transferred into the final product then it cannot be used.

Analysis of the governing equations suggests that ripples will always occur somewhere inside the heater, but numerical solutions indicate that the location and growth rate of these ripples can be manipulated by adjusting the dimensions of the sheet and the temperature profile of the heater. In the future, it may be possible to exploit this fact to ensure ripples do not appear in the redrawn product.



(a) Tensile (red) and compressive (blue) regions in a sheet undergoing redraw. In the blue region, compression acts along the axis indicated by the arrows. (b) Out-of-plane ripples, with compressive region outlined by dashed lines.

Outlook

Our mathematical analysis demonstrates a mechanism for eliminating thick edges from redrawn glass, and this concept is now used to manufacture sheets with close to uniform thickness. We have also gained insight into the formation of out-of-plane ripples, and and uncovered new and challenging mathematics problems associated with studying this.

FURTHER READING

[1] D. O'Kiely, C. J. W. Breward, I. M. Griffiths, P. D. Howell, U. Lange, Edge behaviour in the glass sheet redraw process. J. Fluid Mech. 785 (2015), 248–269.

Doireann O'Kiely

Doireann O'Kiely is a postdoctoral research associate at the University of Oxford. Her PhD was supervised by Chris Breward, Ian Griffiths, Peter Howell, and Ulrich Lange (Schott AG). Her main research interests are in the behaviour of fluid and elastic sheets. Outside of the office she likes to spend time outdoors hiking, running, or in the park with friends.

This section is for Early Career Researchers. Please send suggestions for questions or topics you would like to see covered to newsletter@lms.ac.uk.

Applying for Your First Grant

"Dear X, I'm an early career researcher and I think it's time for me to apply for my first grant. What advice would you give me?" — We invite four experts to comment.

Alison Etheridge is Professor of Probability at the University of Oxford. She has been on both sides of many EPSRC grant panels.

The first thing I would say is don't just apply for a grant because 'it is time'. You need to have a clear idea of a project and how the grant is going to help you complete it. It will be very obvious if you are just doing this because your department wants the overheads, or because you think it will look good on your CV. Usually you will be applying for funds to support a postdoc. Have you thought about where you'll find a suitably qualified candidate? Assuming that the answer is yes, we come to the application.

There are basic points that apply to any application. Funders usually provide very helpful advice on how to lay out your case for support and what they are looking for in each section. Pay attention to this — it marries with the questions asked of the reviewers, who will therefore be looking for answers to those questions in those sections.

Next think about your audience. Your proposal will go to some expert reviewers. Their scores will be the starting point for panel discussions. If you are applying to EPSRC, you can expect that at least one of the reviewers will be from among your own recommendations and, provided your proposal makes sense, they are likely to be supportive. The other reviewers will be chosen by the EPSRC maths team, who will try very hard to find people close to your area. But the team are not mathematicians (that's EPSRC policy) and so the reviewers may be some way away from your own interests. Be careful not to assume too much of the reader; even the more technical sections should be comprehensible to quite a wide range of mathematicians.

Reviewers will expect to see some specific problems (within a bigger programme) and proposed approaches to them. For a first grant they may pay special attention to 'management' of the project — you don't have a long track record of supervision, so be careful to explain how you will go about working with a postdoc and looking after their career development. Evidence of a 'backstop' in the department would be good.

And now think about the panel. Most panel members will only hope to understand the first few paragraphs of your proposal. You have about half a page in which to grab their attention and make them believe that your work is really worthy of support. The question that you absolutely have to answer right at the start is "Why should I care?".

It is easy to dismiss the description for a lay audience. Don't! If you can make your work sound compelling to a non-mathematician, then you'll probably have won over the average panel member. And although lots of the questions (and especially the Gantt chart) can seem really annoying, it is actually a useful exercise to think about the timing of the project and place it in a broader mathematical and scientific context.

Finally, remember that no matter how good your application there is always an element of luck, so you shouldn't be too disheartened if the grant is not awarded. You certainly won't get it if you don't apply. Good luck!

Charlotte Kestner has recently moved to become a teaching fellow at Imperial College. She was a Lecturer at the University of Central Lancashire when she applied for the Leverhulme grant.

I recommend that you talk to your peers and to senior people about your idea, especially people in similar areas who have received grants. It's key that you are ambitious but realistic in your project, and advice from others can help you make sure that your project is pitched at the right level.

I recently got my first grant from the Leverhulme Trust. I had applied for a grant from EPSRC about a year earlier. I didn't get that grant, but I got positive feedback, so I developed the ideas and was delighted to receive the Leverhulme research project grant. Don't be disappointed if you don't get a grant first time, and remember that it's good to apply to many things.

Katharine Moore is a Senior Portfolio Manager at the Engineering and Physical Sciences Research Council (EPSRC), with particular responsibility for Mathematical physics, Mathematical analysis, Fellowships, and Programme Grants.

Your first grant is an important step in your career and can be an excellent way to help build your research profile. However, it can be hard to know where to start when writing your first research proposal. My first piece of advice for anyone who's thinking about applying for their first grant would be to discuss your research ideas and proposal with more senior colleagues. They are likely to raise similar questions to your reviewers, so take their advice on board. Try to ensure that the project has clearly defined objectives and outcomes. These should be articulated clearly both for your reviewers and for EPSRC (or other funder). You should also take advice from your university's research office. They will have seen many research proposals and have experience with different funders.

Aside from the quality of the research, EPSRC proposals are assessed against several other secondary criteria including National Importance and Impact. You should think carefully about how your research addresses these two criteria including reference to EPSRC strategy where appropriate. A good place to start thinking about how your research fits to EPSRC strategy is our Delivery Plan which can be found on our website. A clear pathways to impact statement is required as part of every application to EPSRC. I encourage you to consider how you can accelerate the time it takes for your research results to have benefits beyond the limits of your own area of specialisation. EPSRC consider impact in the broadest sense of the word, breaking it down into the following categories: academic impact, economic impact, societal impact, and impact on people such as training a postdoctoral research assistant or PhD student.

Both EPSRC's Fellowship scheme and New Investigator Award scheme require a letter of support from your host institution and the level of support forms part of the assessment process. It is important that you speak to your Head of Department about how they will support your career development and encourage them to provide specific details of their commitment to you in the letter.

Finally, your PI response to reviewers is important! As panel members are not allowed to re-review proposals, a good response can make a competitive difference.

Iain Stewart is a Professor of Computer Science at Durham University. He has sat on or chaired numerous research funding panels for EPSRC and other international funding organisations. He is currently Programme Secretary for the LMS and for the past three years has chaired the Programme Committee, which decides upon the distribution of research funds for the LMS.

I would advise that when writing your proposal, you work very closely with a mentor who has experience of writing (successful!) grant proposals to the funder in question. There are many subtleties and nuances in writing a good grant proposal, and funding experience and knowledge are absolutely crucial. It is often useful to sit down with your mentor and talk your way through a draft proposal, with the mentor providing constructive criticism and ensuring that you have covered all the angles and interpreted the guidelines appropriately. You might think that finding such a mentor might be tough but many more senior staff will be happy to help; indeed, in any good department such a mentoring scheme should be an expectation. Your mentor does not need to be an absolute expert in the topic of your research; it's the mentor's generic skills that you will hope to access. I would also recommend contacting the funder if there are guidelines that puzzle you as, in my experience, staff who administer funding schemes are almost always very happy to help, especially with those who might be inexperienced in such matters. A short phone call can often prove most useful.

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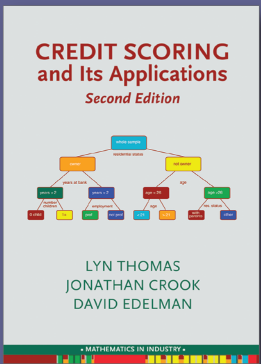

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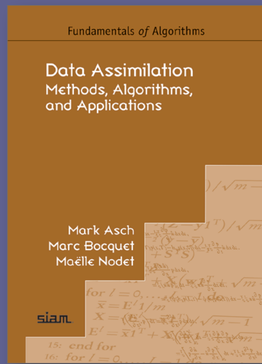

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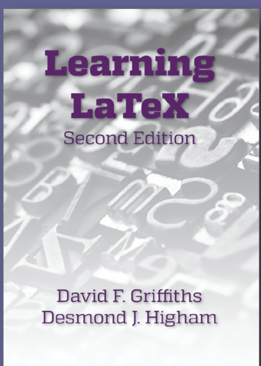

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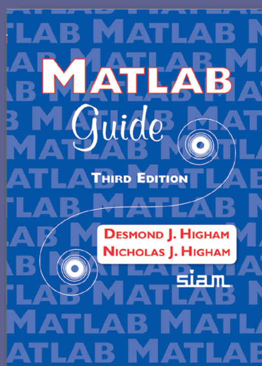

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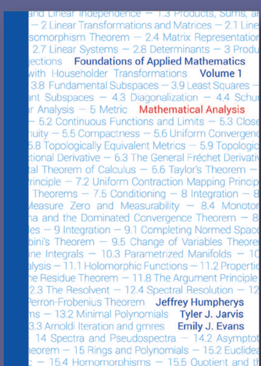

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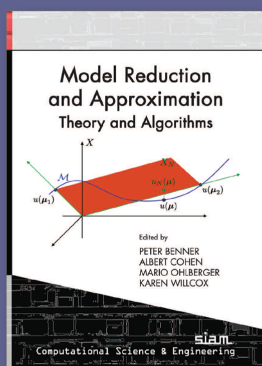

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British Science Festival 2017

Review by Peter Giblin

Sasha Movchan talked on improved treatment of abdominal aneurysm

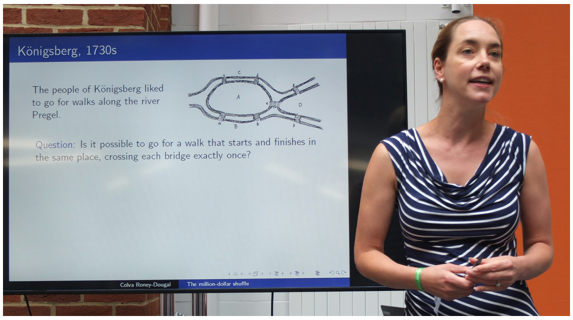
The British Science Association held this year's Festival in Brighton, with events divided between the Falmer campuses of the Universities of Brighton and Sussex, and several city centre venues including, on a rainy Friday evening, the length of Brighton pier. This report is about some events with a significant mathematical content, organized by the Mathematical Sciences Section of the BSA (where I am the IMA representative) or by others, including the Medical Sciences Section. In fact several events had a strong flavour of applications to medical and health sciences, and all emphasized the broad applicability of mathematics.

The 2017 President of the Mathematical Science Section is Colva Roney-Dougal (St Andrews) and her topic was *The million-dollar shuffle: symmetry and complexity*. The Millennial Prize referred to here is for the *P versus NP* problem which was deftly approached via Euler circuits (bridges of Königsberg) and Hamilton circuits (seating feuding family members round circular tables to avoid fights), using symmetry to simplify problems as appropriate. This led into Turing machines, complexity of algorithms — addition, multiplication, factoring — and the distinction between speed of checking a solution and that of finding a solution. A specific example was given of determining hospital rosters, where breaking symmetry may in fact be appropriate; Colva concluded by talking about recent developments in the graph isomorphism problem (László Babai, 2016–17, a 'quasi-polynomial time' algorithm), and quantum computing. Discussion continued long after the lecture, at a drinks reception sponsored by the London Mathematical Society.

The role of mathematics in healthcare, besides design of hospital rosters, was a major topic in three events: Robert Cuffe (head of statistics at BBC News) on *Drugs, condoms and the theory of experimentation*, Hermes Gadêlha (York) on *Male fertility: do the maths*, and Sasha Movchan and Luca Argani (Liverpool) on improved treatment of abdominal aneurysm.

Robert discussed in detail some of the many difficulties of evaluating a drug (specifically, to treat or prevent HIV infection) or comparing one drug with another, difficulties which can be ethical (why you can't undertake a trial in which some patients are given a placebo when they might die as a result), statistical (how do you pick a margin of 'improvement' or distinguish between a failed drug and a failed trial?) or medical (how can you compare drugs when they act in different ways?). He also gave examples of confusing presentation of statistics: given that a certain diet increases the risk of colon cancer by 18% then you need to know also that the baseline risk of this disease is 6%, and understand that the increased risk is therefore $\frac{118 \times 6}{100} = 7.08\%$ and (thankfully) not $18 + 6 = 24\%$.

Hermes talked about the mathematical modelling of sperm mobility. The current way of testing male fertility ('Computer-Aided Semen Analysis' (CASA) system) focusses on simplistic 'sperm counts' and movement of the head, and cannot assess the detailed wiggling of the tail. This means it is able to determine the virility of the sperm but not its ability to actually reach the egg. Hermes's new mathematical developments are providing new tools that can predict how sperm swim by taking into account the movement of the tail, as well. The immediate impact is in the use of clinical predictive tools to assess fertility in clinics. There are other applications too, such as determining how viruses and bacteria with a similar make up are able to move through the body and other liquids. The mathematics is also assisting novel designs of bio-inspired micro-robots. The aim here is to develop 'artificial swimmers' that could carry drugs and achieve biological control at cellular scale, so in essence robotic enabled sperm.



Colva Roney-Dougal spoke on computational complexity

Sasha and Luca described a remarkable application of the theory of waves to the design and use of stents for aneurysms (tubes inserted to bypass dangerous bulges) in the abdominal aorta, one of the main arteries of the body. The volume between the stent and the outer walls of the aneurysm is sealed with a bio-compatible polymer, but the body's natural vibrations can cause separation and migration of the sealant and the expanded artery. Careful analysis of the modes of vibration in three directions has resulted in significantly improved understanding of this process and hence of the treatment's chances of success. This was a collaboration between experts in two radically different fields, mathematics and medicine, and Sasha emphasised the challenge, in the early stages, of finding one's way to a common language. The work was carried out under the auspices of the EPSRC Liverpool Centre for Mathematics in Healthcare. As you can see from the photograph, the lecture was illustrated not only by sophisticated videos of vibrational modes, but also by some low-tech demonstrations!

John Howse (Brighton) took for his topic 'Picturing Problems': the solution of logical puzzles of a familiar form by carefully following through the possibilities using simple diagrams ('Euler diagrams') displaying set inclusion and membership.

A typical (but simple) example is of an island in which everyone is a pianist (P) or a violinist (V) but not both (the usual mathematical meaning of 'or' caused some interesting exchanges with the audience!). Male pianists (MP) always lie, similarly FP always tell the truth, MV always tell the truth and FV always lie. (If only life were that simple.) Then Nell (F) says 'We are both pianists' and Mike (M) says 'That is true.' Then the task is to discover who plays each instrument. The key idea is to use the diagrams to make the chains of logic 'visible'. John concluded by launching into deeper and choppier waters, discussing logical paradoxes and Gödel's incompleteness theorem.

Continuing the theme of visibility, Daniel Colquitt (Liverpool) talked about 'Invisible Mathematics', shorthand for 'cloaking' of a selected region of space by bending microwaves, radio waves, light or even sound waves round the region. Daniel began by emphasizing the wide applicability of a single important idea in applied mathematics, in this case the Helmholtz equation $\nabla^2 A + k^2 A = 0$ which appears in Newton's laws of motion, Maxwell's equations and the propagation of waves. The first realizations of cloaking occurred for microwaves and radio waves; light presents greater difficulty because of the short wavelength, but has even more interesting (or frightening) applications. Actual examples were presented where a square region of a drumhead or a thin plate is cloaked, this shape lending itself well to fabrication of materials. Cloaking of buildings from earthquakes does not appear to be possible but maybe diverting surface seismic waves into body waves penetrating downwards has a better chance of success: experiments are actually in progress in France using a 'metawedge forest', where the trees are regularly planted but of steadily increasing height. Calculations show that this should accomplish the desired diversion of dangerous waves into harmless ones.

In 1886 female students at Vassar College performed *The Mathematikado*, a parody of Gilbert and Sullivan's The Mikado. The festival event by Andrew Fiss and Laura Kasson Fiss (Michigan Technological University) showed, with musical examples and mathematical jokes, how this production responded to contemporary critique of female students participating in mathematics and science. Another musical event by Kelly Snook (Brighton University) demonstrated her work with interactive music inspired by Kepler's investigation of the 'harmony of the spheres'.

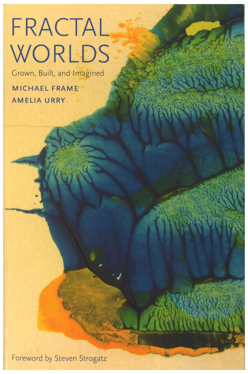
Peter Giblin

Peter Giblin is Professor of Mathematics (Emeritus) at the University of Liverpool. His research interests are in singularity theory and applications to geometry and computer vision. He was until recently chair of the Higher Education Committee of the IMA.

Fractal Worlds

by Michael Frame and Amelia Urry, Yale University Press, 2016, paperback, 536 pages. £18.99, ISBN 9780300197877.

Review by Gavin M Abernethy



This is a very nice, highly readable but detailed introduction to fractal geometry aimed at a non-specialist audience. Coauthored by Michael Frame of Yale and his former student Amelia Urry, it is a very passionate work, with both authors' love for the subject quite apparent. You may grow weary of the references to Mandelbrot and personal anecdotes that permeate the text, but he was clearly of great personal significance to Frame.

The main body varies in mathematical detail from none (Chapter 4) to moderate (Chapter 5 is exclusively a detailed discussion of the Mandelbrot set), with exercises included in several chapters (primarily on iterated function systems and dimension calculations). There is substantially more mathematics than in a popular science work, yet it is much sparser and more readable than a textbook — even in the more technical appendix. Suitable for undergraduates, only a basic familiarity with calculus and algebra is assumed, and real analysis and measure theory are not elaborated on. The book is at least partially based on the course run by Frame at Yale over many years, offering a "visual introduction… accessible to students not majoring in science". However, whilst the written sections should certainly be accessible, a non-scientist would have to be confident in their maths to attempt the exercises.

Nearly half of the text consists of technical notes, which rather than a textbook-style detailing of necessary techniques to study fractals, consists of a selection of examples and outlining ideas that interest the authors but did not make it to the main body. Some proofs are included, but nothing comprehensive.

Most of the main ideas and classic examples of fractal geometry are covered (coastlines, Cantor sets, gaskets, Julia sets, Box-counting and Similarity Dimension), with an emphasis on explaining motivation rather than comprehensive detail. Thus, for example, Hausdorff dimension is not discussed rigorously. Whilst this does rend the text unsuitable for basing any pure mathematics course on it (stick to Falconer's *Fractal Geometry*), it illuminates the main motivations of scaling and self-similarity while allowing us to reach into fairly advanced topics by the final chapters. Chapter 7's sections on multifractals and data-driven iterated function systems were particularly of interest to this reader, especially as the former is not usually included in any detail in an introductory text on this subject.

The most interesting contribution of this text is found in Chapter 4, where the second author's presence is most obviously felt (Urry majored in poetry). This chapter catalogues and offers brief reflections on fractal ideas in anything wrought by humans — capacitors, music, visual art, but most intriguingly, notions of scaling and self-similarity in poetry, literature, narrative structure and meta structure such as the course of movements in the history of art. The examples given are often stimulating, but some are not as well-developed as I would have liked, and none are elaborated on in more than a page or two of detail. This collection is mainly useful to inspire each reader with a different question and give the reference pointing them in the right direction, as of course not all could be pursued in this text. To give one example, a quote from Kate Wilhelm's *Death Qualified* raised the suggestion that fractal geometry's holistic outlook could challenge the popular current of reductionism in contemporary philosophy of science. More existential applications of fractals such as this occasionally occur in other chapters, including an amusing thought exercise of life as it might be perceived if we lived in non-integer dimensions.

Furthermore, these chapters indicate a possible use of the text for specialists and lecturing staff: do you want to inspire and motivate your class with suggestions of fractal applications to unusual topics? You will almost certainly find something here that will suffice, or provide a jumping-off point for a student project on fractal antennae, earthquakes, or the distribution of shot durations in feature films for example.

Apparently not mathematical Platonists ("Indeed, all of mathematics, all of science, lives in our heads." Pg.108), the authors emphasise that our current paradigms of, for example, manufacturing technique are based on the early emergence of Euclidean geometry when it need not necessarily have been so. Thus they hope that the alternative methods of fractal geometry should reawaken a childlike inquisitiveness in the reader. As Frame puts it in an unexpectedly moving Valediction, these tools provide a "path to return to the wonder of childhood, deepest of all joys" (Pg.252). Though they may be overstating the case for fractals as a new "theory of everything", their palpable excitement embedded throughout the text is admirable and conveys a strong motivation for studying the subject to the non-specialist reader.

In summary, this work is enthusiastic, varied, and thought-provoking. Your experience of the more speculative writing may vary with personal taste, and a firmer hand at the editing stage would not have gone amiss. It is heavier in some chapters than many popular science texts, with exercises included, but highly readable nonetheless. It would be particularly suitable for instructors looking for a pedagogical aid or a catalogue of ideas and inspirational material, or for a confident non-specialist seeking a serious introduction that strives to embed the subject in the natural and artistic world.

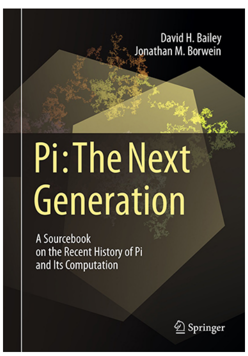
Gavin Abernethy

Gavin Abernethy is a PhD candidate in the School of Computing and Mathematics at Ulster University. He obtained his undergraduate degree in Mathematics at the University of St Andrews, with an emphasis on abstract analysis and measure theory. Current research interests include computational ecology and evolutionary food web models. He enjoys computer games and Tesco "everyday value" midget gems (25p a bag).

Pi: The Next Generation

by David H. Bailey and Jonathan M. Borwein, Springer, 2016, hardback, xiv+507 pages. £52.99, ISBN 9783319323756

Review by Thomas Sonar



Of all the 'magic numbers' known to mankind, starting with the number 666 of St John's Apocalypse, arguably none has fascinated so many people over so many centuries as π. Among other things, the nature of π lay at the foundation of the problem of squaring the circle and it was not before the second half of the 19th century that Ferdinand Lindemann could prove that π in fact is a transcendental number. Hence when Berggren, Borwein and Borwein published *Pi: A Source Book* in 1997 it immediately became a success and today is available in its third edition. While *Pi: A Source Book* contained mainly reprints of classical papers on the nature of π (starting with the Rhind papyrus), the companion volume, *Pi: The Next Generation* is compiled as a sourcebook on the recent history of π from 1975 on, and on computational issues.

The book contains the reprints of 25 papers which are introduced by short summaries of their contents. The first five papers in this new book on the number π are concerned with the computation of π using the arithmetic-geometric mean as conceived by Carl Friedrich Gauss. In particular, the third paper by David Cox gives a thorough introduction to the AGM. With paper 6 by Stan Wagon the discussion of a fairly modern conjecture concerning π begins: Is π normal? (This paper by Wagon was also included in the older book). The question of normality was raised in 1906 by Borel and is still not answered today. A real number is said to be *normal in base* b, if in its representation in base b all digits occur equally often in an asymptotic sense. Hence the question is: Do the digits in the representation of π appear 'at random'? Already Borel proved that the set of non-normal numbers has measure zero; however, it is not easy to give concrete examples of normal numbers. In papers 17 and 23 normality is again the issue. It is fascinating that an easy question such as normality still can not be answered in the case of π.

Many articles in the book are concerned with fast algorithms for the computation of the digits in the decimal representation of π, e.g. papers 5, 7, 9, 13, 14, 16, 17, 20, 22. Others emphasize the role of π in the history of mathematics; for example paper 8 on Gauss, Landen and Ramajunan by Almkvist and Berndt; paper 10 on Ramanujan and π by Borwein and Borwein; paper 11 on Ramanujan, modular equations and approximations to π by Borwein, Borwein and Bailey, where quadratically convergent algorithms for the computation of digits are also discussed. Interesting and fascinating hidden properties of π are also discussed. In paper 12 on π, Euler numbers, and asymptotic expansions by Borwein, Borwein and Dilcher the interesting phenomenon is the following: Using Gregory's series

$$\pi = 4\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{2k-1}$$

and summing up to 500 000 terms, one finds that the sum is incorrect in the sixth digit (not surprisingly), but the following ten digits are correct (surprisingly!). This pattern even occurs further on. The careful analysis provided gives deep insight into the nature of the decimal expansion of π. In paper 15 Huylebrouck examines interesting similarities in the irrationality proofs for π, $\ln 2$, $\zeta(2)$, and $\zeta(3)$. In paper 18 Lucas shows an interesting connection between certain integrals of positive integrands and approximations to π; in paper 19 Baruah, Berndt and Chan examine Ramanujan's series for $1/\pi$. There is also a hidden link between the representation of π and random walks, which again leads to the question of normality of π. This link is shown in paper 21 where Artacho, Bailey, Borwein and Borwein are 'Walking on real numbers'.

In paper 24 J.M. Borwein gives a panorama of π through the ages, 'The Life of π: From Archimedes to ENIAC and Beyond'. The final paper, 'I prefer π: A brief history and anthology of articles in the American Mathematical Monthly' of 2015 presents a brief summary of papers on π which appeared in the *Monthly* and is well-chosen to conclude the present volume.

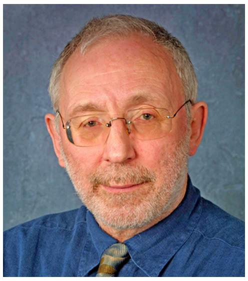
The present sourcebook does not depend on the older book by Berggren, Borwein and Borwein, although reading the latter can provide valuable insights into the older history of π. The papers included are varying in mathematical depth and, not surprising in a book edited by Jonathan Borwein, clearly show the important role of 'Experimental Mathematics', i.e. exploring mathematics by means of ingenious computer algorithms. Reading the papers in this book I found many aspects on the mathematics and history of π which I did not know before and I enjoyed reading it very much. As the older book on π this one will also soon become a standard reference tool for working mathematicians and historians of mathematics alike.

Thomas Sonar

Thomas Sonar is a professor of mathematics at the Technical University of Brunswick in Germany. His main research interests are the numerical analysis of hyperbolic conservation laws and the history of mathematics, in particular the history of analysis and of numerical methods. Thomas was born in Germany but retains strong links to Oxford where he worked on his PhD thesis under Keith William (Bill) Morton ages ago and where he returns regularly conducting research on the history of mathematics.

Obituaries

John Elgin: 1946 – 2017



John Elgin, who was elected a member of the LMS on 20 November 1987, died on 17 July 2017, aged 70.

John Gibbon and Chris Eilbeck write: John was born into a south-side Edinburgh tenement family. He attended a local School, commonly known as 'Jimmy's', where expectations were low, and left at 15 with no qualifications. Joining Parsons Peebles Turbines as an apprentice, his potential was soon realised and he was encouraged to study at night school, first for an ONC, and then to study Physics at Heriot-Watt University. Graduating with a First, and, following a summer at CERN in Geneva, he then joined the PhD programme at DAMTP in Cambridge in 1970. Attached to Corpus Christi College and funded by a Carnegie Scholarship, he pursued thesis work on surface waves at vacuum-plasma interfaces supervised by Philip Clemmow.

In 1974 he entered Imperial College's Physics Department as a postdoctoral research fellow working with Geoffrey New on theoretical nonlinear optics. In 1978 he was awarded a Research Council five-year Advanced Fellowship but then, in 1983, he switched to the Maths Department as a 'New Blood' lecturer. Promotion to Reader followed in 1991, to Professor in 1995, followed by a stint as Section Head of Applied Mathematics (1998-2003) and finally as Head of Department (2003-08). He also served on the advisory panels for Edinburgh's ICMS (2005-) and the Edinburgh Research Partnership (2006-). For the period 2011-12 he was also an LMS MARM (Mentoring African Research in Mathematics) facilitator, working on the partnerships between European and Sub-Saharan institutions, maintaining contacts with mentors and mentees, and seeking new sponsorship for the Project.

John started research at a time when the new science of 'nonlinear systems' was becoming established. Among his wide interests, his work on nonlinear optics led him to write a string of papers addressing the problem of energetic pulse propagation in optical fibres. During his career, he published almost 100 scientific papers. He was a popular lecturer at all levels and guided a string of PhD students, several to academic careers.

Together with Kathy, his wife of 46 years, he shared a strong interest in the arts world and a love of travel. His easy, gentle manner and dry Scottish humour will be missed by everyone.

Cathleen S. Morawetz: 1923 – 2017

Cathleen Synge Morawetz, who was elected a member of the London Mathematical Society on 22 June 2001, died on 8 August 2017, aged 94. Cathleen was the first female LMS Honorary Member and in 2001 the Mary Cartwright Lecturer.

Susan Friedlander writes: Cathleen was very much a "people" person with great charm and distinction. Through her research and her academic leadership she influenced generations of academics. Her father was J.L. Synge, a distinguished Irish mathematician who had a long career at the University of Toronto. Cathleen credited her father with instilling the ideal of intellectual achievement and her mother with instilling ambition. After an undergraduate degree at the University of Toronto and a masters degree at MIT, Cathleen came to the Courant Institute at NYU where she stayed for her whole career. She moved through many positions, beginning as a research assistant for Richard Courant and moving up the academic ladder to become the Director of the Institute. Influenced by the groundbreaking work of Richard Courant and Kurt Friedrichs, the focus of Cathleen's research was originally on transonic flow and shock waves and later the more general area of nonlinear waves. Her work in analysis introduced a useful tool that is now referred to as Morawetz inequalities.

Cathleen's achievements and leadership talents were recognized by many awards and distinctions. These included a very impressive set of "first for a woman". In 1981 she was the first woman to give the AMS Gibbs Lecture. In 1984 she was the first woman to be appointed Director of the Courant Institute or any comparable directorship within the mathematical sciences in the US. In 1990 she was the first woman to be elected to the Applied Mathematics Section of the US National Academy of Sciences. In 1998 she was the first woman in the mathematical sciences to be awarded the National Medal of Sciences, the highest scientific honor the US can give. The citation for this award says it was given to Morawetz "for pioneering advances in partial differential equations and wave propagation resulting in applications to aerodynamics, acoustics and optics". In 2004 she was the first woman to receive the AMS Steele Prize for Lifetime Achievement and in 2006 she was the first woman to receive the AMS-SIAM Birkhoff Prize in Applied Mathematics. She was not the first but the second woman to become President of the AMS in 1995. We note that it is not until this year of 2017 that a third woman will be elected AMS President.

The list of honors which Cathleen received certainly does not stop at those already mentioned above. She has been awarded honorary degrees by Eastern Michigan University, Smith College, and Brown University in 1982; Princeton University in 1986; and Duke University, and New Jersey Institute of Technology in 1988. In 1993 she was named Outstanding Woman Scientist by the Association for Women in Science. In 1997 she received the Krieger-Nelson Award from the Canadian Mathematical Society. She has been elected a Fellow of the American Association for the Advancement of Science and she was elected a Member of the American Academy of Arts and Sciences.

Alasdair Rose: 1949 – 2017

Dr Alasdair E.A. Rose, who was elected a member of the London Mathematical Society on 14 May 2003, died on 17 June 2017, aged 68.

Philippa Hemmings writes: Dr Rose worked for Science and Engineering Research Council, and then the Engineering and Physical Sciences Research Council for more than 30 years. I first met him when I joined EPSRC as a Committee Secretary and Alasdair was Head of Chemistry and Physics at SERC and was struck by his passion for scientific research across all fields. He was EPSRC's second Programme Manager for Mathematical Sciences between 1998 and 2003 and brought dedication and enthusiasm to the role and a strong desire to support UK mathematical sciences. He was awarded an MBE in the Queen's New Year's Honours in 2000 for services to scientific administration. Alasdair subsequently took on the role of head of the newly created Basic Technology Programme and was appointed to a new role as Head of Economic Impact in 2006. He continued to retain a strong interest in the mathematical sciences.

After retiring from EPSRC in 2013, he returned to Scotland with his family. Colleagues who worked with him at EPSRC remember him with affection and respect for his kindness and integrity.

LMS Meeting

Annual General Meeting of the LMS

10 November 2017, 3–6pm. BMA House, Tavistock Square, London WC1H 7JP

Website: lms.ac.uk/events/society-meetings

The meeting will open with a brief introduction and a presentation on Society Business. This will be followed by a lecture by **Zoubin Ghahramani** (Cambridge University; Chief Scientist, Uber) on *Bayesian statistics, non-parametrics, neural networks, and artificial intelligence*, and a presidential address by LMS President **Simon Tavaré** (Department of Applied Mathematics and Theoretical Physics & Cancer Research UK Cambridge Institute, Cambridge University) on *The magical Ewens sampling formula*.

These lectures are aimed at a general mathematical audience. All interested are most welcome to attend.

The meeting will include the presentation of certificates to all 2017 LMS prizewinners and the announcement of the Annual LMS Election results.

The meeting will be followed by a reception, which will be held at De Morgan House, 57-58 Russell Square, London, WC1B 4HS.

For further details about the AGM, please contact Elizabeth Fisher (lmsmeetings@lms.ac.uk)

The Society's **Annual Dinner** will also be held on 10 November at 7.30 pm at the Montague on the Gardens, 15 Montague St, Bloomsbury, London WC1B 5BJ. The cost of the dinner will be £58, including drinks. To reserve a place at the dinner, please email John Johnston (john.johnston@lms.ac.uk).

LMS Meeting

Graduate Student Meeting

10 November 2017, 10am–3pm. BMA House, Tavistock Square, London WC1H 7JP

Website: lms.ac.uk/events/society-meetings

Dr Ioanna Manolopoulou (UCL)
Clustering Variable-Length Order Statistics in Retail Analytics

Dr Hao Ni (UCL)
Modelling the Effects of Data Streams using the Rough Paths Theory

These lectures are aimed at a general mathematical audience. All interested, whether LMS members or not, are most welcome to attend this event.

The meeting will include student presentations of their current work, with a prize awarded for the best student talk.

The meeting will be followed by the LMS Annual General Meeting and a reception, which will be held at De Morgan House, 57-58 Russell Square, London, WC1B 4HS.

Travel grants of up to £50 are available for students who attend both the Graduate Student Meeting and the LMS Annual General Meeting.

For further details about the GSM, please contact Anthony Byrne (lmsmeetings@lms.ac.uk)

LMS Annual Dinner
The Society's Annual Dinner will also be held on 10 November at 7.30 pm at the Montague on the Gardens, 15 Montague St, Bloomsbury, London WC1B 5BJ.

The cost of the dinner will be £58, including drinks. To reserve a place at the dinner, please email John Johnston (john.johnston@lms.ac.uk).

LMS Meeting

South West & South Wales Regional Meeting

13 December 2017, 3 pm, Cardiff

Website: lqp2.org/lms17

Shahn Majid (Queen Mary University, London)
Braided algebra and dual bases of quantum groups

Ingo Runkel (Hamburg)
Categorification and field theory

These lectures are aimed at a general mathematical audience.

A Graduate Student Meeting will also take place in the morning on 13 December 2017.

All interested, whether LMS members or not, are most welcome to attend these events.

The meeting forms part of a workshop on *Algebraic Structures and Quantum Physics* from 13 to 15 December 2017. For further details visit: lqp2.org/lms17 or contact the organisers (LechnerG@cardiff.ac.uk, PughMJ@cardiff.ac.uk, WoodSi@cardiff.ac.uk).

There are funds available to contribute in part to the expenses of members of the Society or research students to attend the meeting and workshop. Requests for support, including an estimate of expenses, may be addressed to the organisers.

Registration and Society Dinner

For further details and to register and to reserve a place at the dinner, please visit lqp2.org/lms17. The cost of the dinner will be approximately £40, including drinks.

Spectral Theory of Hankel Operators Research Workshop

Location:	King's College London
Date:	2–3 November 2017
Website:	tinyurl.com/Hankel2017

This is a two days EPSRC supported research workshop. The aim is to bring together mathematicians working on different aspects of spectral theory of Hankel operators. See the list of speakers on the website.

Contact: Alexander Pushnitski (King's College London).

NBFAS in 2017/18

Location:	University of Leeds
Date:	10 and 11 November 2017
Location:	University of Newcastle
Date:	28 and 29 May 2018
Website:	tinyurl.com/y99jw6w4

Speakers at the two regular meetings can be seen on the website. A special meeting to celebrate the 50th anniversary of NBFAS will be held from 12 to 14 April 2018 at the University of Edinburgh. NBFAS is partly supported by an LMS Scheme 3 grant. The anniversary meeting is supported by an LMS Conference grant.

Integrable Day at Loughborough

Location: Loughborough University
Date: 24 November 2017
Website: tinyurl.com/y8uuu3se

A half-day workshop on *Modular Forms and Applications* is part of a collaborative workshop series on Classical and Quantum Integrability, involving Edinburgh, Glasgow, Heriot-Watt, Leeds, Loughborough and Northumbria Universities. Funds may be available to support the attendance of research students. Enquiries should be addressed to the organisers: E.V.Ferapontov@lboro.ac.uk and A.P.Veselov@lboro.ac.uk. The workshop is supported by an LMS Scheme 3 grant.

2018 British Postgraduate Model Theory Conference

Location: Mathematical Institute, Oxford
Date: 4 – 6 January 2018
Website: tinyurl.com/yat5l4jn

The aim of this conference is to provide a platform for postgraduate students working in model theory to present their research. Register via the above website by 3 January 2018. If you would like to give a talk please email the address given on the website by 4 December 2017.

Supported by the LMS through a Postgraduate Conference grant.

Scalable Statistical Inference Day

Location: University of Sussex
Date: 25 January 2018
Website: tinyurl.com/y9ydoksg

The area of computational statistics is undergoing a phase of rapid development, aided by new ideas coming from probability and applied mathematical analysis. This workshop aims to explore recent results in this area, with talks given by Michela Ottobre (Heriot-Watt), Lukasz Spruch (Edinburgh/ATI) and Andrew Duncan (Sussex/ATI). This workshop forms part of a three day event, kindly supported by an LMS Scheme 1 grant to celebrate recent appointments in the Mathematics Department at the University of Sussex.

Functor Categories for Groups

Location: Royal Holloway, University of London
Date: 8 December 2017
Website: tinyurl.com/y7wcpmn6

Cohomology of functor cateogries and classifying spaces is the third meeting of the Research Group Functor Categories for Groups (FCG). We shall focus on cohomological finiteness conditions for discrete as well as totally disconnected locally compact groups. FCG Research Group is supported by an LMS Joint Research Groups in the UK Scheme 3 grant. Limited funding is available for PhD students, allocated on a first come first served basis. To register for the event, email the local organiser Brita Nucinkis (brita.nucinkis@rhul.ac.uk).

Mathematics in Materials Science

Location: University of Sussex
Date: 24 January 2018
Website: tinyurl.com/y7x6ajmf

This one-day meeting aims to bring together researchers in the UK who work on the interface between mathematics and materials science with talks given by Professor John Ball (University of Oxford), Dr Lucia Scardia (University of Bath) and Dr Konstantinos Koumatos (University of Sussex).

The meeting is part of a three day event supported by an LMS Celebrating New Appointments Scheme 1 grant.

Day on Markov Chains

Location: University of Sussex
Date: 26 January 2018
Website: tinyurl.com/y7uqv863

The meeting aims to reflect recent advances in the asymptotic theory of Markov chains, with a particular emphasis on the delicate subject of chains with vanishing drift. There will be talks given by Denis Denisov (Manchester), Dmitry Korshunov (Lancaster) and Vladislav Vysotsky (Sussex).

The meeting is part of a three day event supported by an LMS Celebrating New Appointments Scheme 1 grant.

Indra's Pearls: A Mathematical Adventure

Location: University of Lincoln
Date: 7 February 2018
Website: tinyurl.com/y8f9vnoy

The Annual Charlotte Scott Lecture in Mathematics will be given by Professor Caroline Series FRS on *Indra's Pearls: A Mathematical Adventure*. This public lecture is a part of celebrations of the 160th anniversary of Charlotte Scott, the famous mathematician born in Lincoln, who was also influential in developing mathematical education of women and their participation in mathematical research.

2017 David Crighton award: presentation and lecture

Location: Royal Society
Date: 15 March 2018

The 2017 David Crighton Medal will be presented to Professor I. David Abrahams on Thursday 15 March 2018 at the Royal Society. The presentation will be followed by a talk by Professor Abrahams, and a reception. Details and booking information to follow.

Probability, Analysis and Dynamics '18

Location: University of Bristol
Date: 4 – 6 April 2018
Website: tinyurl.com/ybguxrrx

PAD'18 is a three-day conference supported by the Heilbronn Institute for Mathematical Research, consisting of invited talks accessible to researchers in all three fields. Register by 21 March 2018 (fee: £50 to cover catering costs). Places are limited. Details of available funding on website.

Modelling in Industrial Maintenance and Reliability

Location: Manchester Conference Centre
Date: 13 – 15 June 2018
Website: tinyurl.com/ydxw48z4

This 10th IMA international conference is the premier maintenance and reliability modelling event in the UK and builds upon a very successful series of previous conferences. Submit abstracts of 100 to 200 words via my.ima.org.uk by 1 February 2018.

Mary Cartwright Lecture

Location: LMS, De Morgan House
Date: 2 March 2018
Website: tinyurl.com/ycv3ssfq

The Mary Cartwright Lecture forms part of the annual programme of LMS Meetings, for which the Women in Mathematics Committee selects the speakers. See website for details of speakers. The meeting will be followed by a wine reception. To register contact John Johnston (john.johnston@lms.ac.uk). Attendance is free but numbers are required for catering purposes.

Statistics of Geometric Features and New Data Types

Location: Isaac Newton Institute, Cambridge
Date: 19 – 23 March 2018
Website: tinyurl.com/ybbr89dx

This workshop will be an opportunity for practitioners to learn about the challenges faced by Big Data practitioners, and for mathematically-oriented scientists to discover the most important new data types that can shape their research agendas. Closing date for applications: 21 January 2018.

Nonlinear Analysis and the Physical and Biological Sciences

Location: ICMS, Edinburgh
Date: 21 – 22 May 2018
Website: tinyurl.com/nlanalysis

A Workshop will be held in memory of Professor Jack Carr. A full list of speakers is on the website. Registration will open early in 2018 via the ICMS website. Financial support has been provided by EMS, GMJT, Heriot-Watt University, ICMS, the Maxwell Institute for Mathematical Sciences, and NBDES.

International Congress of Mathematicians 2018

Location: Rio de Janeiro
Date: 1 – 9 August 2018
Website: tinyurl.com/y8jhnbhg

The Organizing Committee has released the first part of the Scientific Programme for the International Congress of Mathematicians: see website. The remaining part of the programme will be released later on. Registration for the Congress and submission of short communications and posters is now open.

Society Meetings and Events

November

2 BCS-FACS Evening Seminar: joint event with the LMS, London
10 Graduate Student Meeting, London
10 Society and Annual General Meeting, London

December 2017

13 SW & South Wales Regional Meeting, Cardiff

March 2018

2 Mary Cartwright Meeting, London

Calendar of Events

This calendar lists Society meetings and other mathematical events. Further information may be obtained from the appropriate LMS Newsletter whose number is given in brackets. A fuller list is given on the Society's website (www.lms.ac.uk/content/calendar). Please send updates and corrections to calendar@lms.ac.uk.

November 2017

1-2 Categorical Methods in Mirror Symmetry, University of Kent (472)
2 What is Mathematics Education, Really?, Lincoln (472)
2 BCS-FACS Evening Seminar: joint event with the LMS, London (472)
2 Symbolic Computation Techniques in SMT
2-3 Spectral Theory of Hankel Operators Research Workshop, King's College London (473)
3 Combinatorics and Computation in Groups, ICMS, Edinburgh (482)
6-10 Ice-Structure Interaction INI Workhop, Cambridge (471)
7-8 Opportunities for the Future: Women in Mathematics, Bristol
10 Graduate Student Meeting, London (473)
10 Society and Annual General Meeting, London (473)
10-11 North British Functional Analysis Seminar, Leeds (473)
11-12 MathsJam Annual Gathering, Yarnfield Park, Staffordshire (472)
13-17 Shape Analysis and Computational Anatomy INI Workshop, Cambridge (471)
24 Integrable Day, Loughborough (473)
29-1 Dec Form and Art, Toys, and Games INI Workshop, Cambridge (471)

December 2017

4-8 New Advances in Fano Manifolds, INI Workshop Cambridge (472)
4-8 Ice Fracture and Cracks INI Workshop, Cambridge (472)
8 Functor Categories for Groups, Royal Holloway, University of London (473)
11-14 Flows, Mappings and Shapes INI Workshop, Cambridge (472)
11-15 Second joint mathematical meeting Spain-Brazil
13 SW & South Wales Regional Meeting, Cardiff (473)
18 Variational Approaches to Problems in Solid Mechanics, University of Warwick (472)
18-22 A Random Event in Honour of Ilya Goldsheid's 70th Birthday, Queen Mary, University of London (472)

January 2018

- 4-6 British Postgraduate Model Theory Conference, Oxford (473)
- 11-12 Two Nonlinear Days, Perugia
- 15-19 Theoretical and Algorithmic Underpinnings of Big Data INI Workshop, Cambridge (472)
- 24 Mathematics in Materials Science, Sussex (473)
- 25 Scalable Statistical Inference Day, Sussex (473)
- 26 Day on Markov Chains, Sussex (473)

February 2018

- 7 Indra's Pearls: A Mathematical Adventure, Lincoln (473)

March 2018

- 2 Mary Cartwright Lecture, London (473)
- 19-23 Statistics of Geometric Features and New Data Types, INI Workshop, Cambridge (473)

April 2018

- 3-6 British Congress of Mathematics Education, Warwick (471)
- 4-6 Probability, Analysis and Dynamics '18, Bristol (473)
- 12-14 North British Functional Analysis Seminar, Edinburgh (473)

May 2018

- 21-22 Nonlinear Analysis and the Physical and Biological Sciences, Edinburgh (473)
- 28-29 North British Functional Analysis Seminar, Newcastle (473)

June 2018

- 11-14 British Mathematical Colloquium 2018, University of St Andrews (472)
- 13-15 Modelling in Industrial Maintenance and Reliability, Manchester Conference Centre (473)

July 2018

- 13-15 From the Foundations of Simulation to Quasi Monte Carlo, LMS Invited Lectures, Warwick (473)

August 2018

- 1-9 International Congress of Mathematicians, Rio de Janeiro (473)

September 2018

- 3-7 Dynamics Days Europe, Loughborough

December 2018

- 11-14 Spain–Brazil Joint Meeting, Cádiz, Spain